Question Bank

CC-12: Group Theory-II & Linear Algebra-II

Unit-1: Group Theory-II

Questions carrying 5 marks

1. Prove that $Z_m \times Z_n$ is cyclic if and only if $gcd\,(m, n) = 1$. Is $Z \times Z$ cyclic? Justify. **CO 2**

2. Show that for any prime p, there exist only two non-isomorphic groups of order p^2. **CO 1**

3. (i) If G is the internal direct product of $N_1, N_2, ..., N_k$ and if $a \in N_i$, $b \in N_j$ for $i \neq j$, then prove that $N_i \cap N_j = \{e\}$ and $ab = ba$. **CO 2**
   (ii) Let p, q be odd primes and let m and n be positive integers. Is $U(p^m) \times U(q^n)$ cyclic? Justify. Here $U(n)$ denotes the group of units modulo n.

4. Show that S_3 has a trivial centre and it can not be expressed as an internal direct product of two non-trivial subgroups. **CO 2**

5. i) State fundamental theorem of finite abelian groups. **CO 1**
   ii) Find all abelian groups (up to isomorphism) of order 360.

Questions carrying 3 marks

1. If $Z(G)$ be the centre of a group G, then prove that $G/\,Z\,(G) \cong Inn(G)$. **CO 1**

2. Exhibit an automorphism of Z_6 that is not an inner automorphism. **CO 1**

3. If an abelian group G is the internal direct product of its subgroups H and K, then prove that$H \cong G/K$ and $K \cong G/H$. **CO 2**

4. Show that the Klein 4-group is isomorphic to the direct product of a cyclic group of order2 with itself. **CO 2**

5. If G is a non-commutative group, then prove that G has a non-trivial automorphism.

Multiple Choice Questions (2 marks)

1. Largest order among the elements of $Z_{30} \times Z_{20}$ is **CO 2**
   (i) 30 (ii) 20 (iii) 60 (iv) 10

2. Let G be a group of order 77 and a be an element of G of order 7. The number of conjugates of a is : **CO 1**
   (i) 1 (ii) 7 (iii) 6 (iv) 77

3. Let G be a cyclic group of order 40. Then which one of the following is true? **CO 2**
   (i) $G \cong Z_2 \times Z_{20}$ (ii) $G \cong Z_4 \times Z_{10}$ (iii) $G \cong Z_8 \times Z_5$ (iv) $G \cong Z_{20} \times Z_2$

4. Number of non-isomorphic abelian groups of order $(2017)^3$ is **CO 1**
   (i) 1 (ii) 2017 (iii) 3 (iv) 3×2017

5. Number of automorphisms on $Z_2 \times Z_2$ is **CO 2**
   (i) 1 (ii) 6 (iii) 4 (iv) 8

6. Let G be a cyclic group of order 2021. Then the number of automorphisms defined on G is **CO 1**
   (i) 2020 (ii) 1932 (iii) 1 (iv) 1680.

7. If G be an infinite cyclic group, then the Aut (G) is a group of order **CO 1**
   (i) 1 (ii) 2 (iii) 3 (iv) infinite.

8. Let G be a group and $f : G \rightarrow G$ be an automorphism such that $f(x) = x^n$ where n is a fixed integer. Then **CO 1**
   (i) G is commutative
   (ii) $a^n \in Z(G)$ for all $a \in G$
   (iii) $a^{n-1} \in Z(G)$ for all $a \in G$
   (iv) none of these

Answer: 1. iii) 2. i) 3. iii) 4. iii) 5. iii) 6. i) 7. ii) 8. iii)

CC-12: Group Theory-II & Linear Algebra-II

Unit-2: Linear Algebra-II

Questions carrying 5 marks

1. Prove that any two matrix representations of a bilinear form are congruent. **CO 3**

2. Diagonalise the symmetric matrix A = $\begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$. has a Jordan canonical form. Find a Jordan canonical **CO 4**

3. Show that the matrix A = $\begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{pmatrix}$ has a Jordan canonical form. Find a Jordan canonical form of A. What are the number of distinct Jordan canonical forms of A? **CO 4**

4. Use Gram–Schmidt orthonormalization process to find an orthonormal basis of R^3 from the basis {(1, 0, 1), (1, 1, 1), (1, 3, 4)}. **CO 3**

5. Reduce the equation $7x^2 - 2xy + 7y^2 -16x + 16y -8 = 0$ into canonical form and determine the nature of the conic. **CO 4**

Questions carrying 3 marks

1. Let $f(x,y) = x^2 + y^2 + xy$. Find the Hessian matrix of f at (0,0) and so that f has a local minimum at (0,0). **CO 3**

2. Show that the sum of two inner products is again an inner product. **CO 3**

3. Let W be the subspace of R^3 spanned by (1, 1, 0) and (0, 1, 1). Find a basis of the annihilator of W. **CO 3**

4. Obtain the eigenvalues, eigenvectors and eigenspaces of the matrix A = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **CO 4**

5. Find all possible Jordan canonical forms for the matrix whose characteristic polynomial is $(t-2)^4(t-5)^3$ and minimal polynomial is $(t-2)^2(t-5)^3$. **CO 4**

Multiple Choice Questions (2 marks)

1. Let $T: R^2 \to R^2$ be a linear functional defined by $T(a, b) = (2a + b, a - 3b)$ and T^* be the adjoint of T, then $T^*(3, 5)$ is equal to **CO 3**
   i) $(6, -5)$ ii) $(11, -12)$ (iii) $(0, 0)$ (iv) none of these

2. Let $\{u_1, u_2, u_3, u_4, u_5\}$ be an orthogonal basis of R^5, y be a vector in R^5, W_1 = Span $\{u_1, u_2\}$, W_2 = Span $\{u_3, u_4, u_5\}$. If $W^\perp$ denotes the orthogonal complement of W, then which of the following is false? **CO 3**
   i) $W_1 = W_2^\perp$
   ii) $W_2 = W_1^\perp$
   iii) there are two vectors Z_1 in W_1 and Z_2 in W_2 such that $y = Z_1 + Z_2$
   iv) y is orthogonal to W_1 and to W_2

3. Let $T: R^2 \to R^2$ be a linear transformation such that $T(1, 2) = (2, 3)$, $T(0, 1) = (1, 4)$. Then $T(5, 6)$ is **CO 3**
   i) $(6, -1)$ ii) $(-6, 1)$ iii) $(-1, 6)$ iv) $(1, -6)$

4. Let A be a matrix of the quadratic form $(x_1 + 2x_2 + \dots + nx_n)^2$, then the sum of the entries of A is : **CO 3**
   i) $\sum n$ ii) $\sum n^2$ iii) $\sum n^3$ iv) $\sum n^4$

5. If the quadratic form $x^2 + \lambda(y^2 + z^2) + 2xy$ is positive definite , then **CO 4**
   i) $\lambda = 5$ ii) $\lambda > 1$ iii) $\lambda < 1$ iv) $\lambda = 2$

6. The minimal polynomial of the matrix $\begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$ is **CO 4**
   i) $(x + 1)(x - 2)$ ii) $(x - 1)(x - 2)$ iii) $(x - 1)(x - 2)^2$ iv) $(x + 1)^2(x - 2)$

7. Signature of the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ is **CO 3**
   i) 1 ii) 2 iii) 3 iv) 4

8. The real quadratic form $ax^2 + 2hxy + by^2$ is positive definite, if **CO 3**
   i) $a > 0$
   ii) $h^2 > ab$
   iii) $a < 0$ and $ab < h^2$
   iv) $a > 0$ and $ab > h^2$

Answer: 1. iv) 2. iv) 3. i) 4. iii) 5. ii) 6. ii) 7. iii) 8. iv)

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CBCS; B.Sc. (H) Mathematics

Question Bank

CC13: METRIC SPACE & COMPLEX ANALYSIS

UNIT 1 (METRIC SPACES)

Questions carrying 5 marks

1. Prove that $diam\ (A) = diam(\overline{A})$ for any non empty subset of a metric space. **CO 1**

2. State and prove Cantor Intersection Theorem. **CO 2**

3. Prove that set of real numbers with usual metric has no proper clopen subsets. **CO 1**

4. Let $\{x_n\}$ and $\{y_n\}$ be sequences in a metric space (X, d). Write $a_n = d(x_n, y_n)$ for all $n \in \mathbb{N}$. If $\{x_n\}$ is a Cauchy sequence and $a_n \to 0$ with respect to usual metric on $\mathbb{R}$, then prove that $\{y_n\}$ is a Cauchy sequence. Is this true if $\{a_n\}$ converges to nonzero limit? Justify. **CO 1**

5. Prove that a point x is a limit point of a set A in a metric space (X,d) iff every ball centred at x contain infinitely many points of the set A. **CO 1**

6. State and prove Heine-Borel Theorem. **CO 2**

7. Show that the derived set and closure of any set is always closed in a metric space. **CO 1**

8. Prove that interior of the set A is the largest open set contained in A. **CO 1**

9. Prove or disprove : Let (X, d) be a metric space and A be a closed and bounded subset of X. Then A is compact. **CO 1**

10. Let (X, d_1) and (Y, d_2) be two metric spaces and f : (X, d_1) → (Y, d_2) be uniformly continuous. Show that if $\{x_n\}$ is a Cauchy sequence in (X, d_1) then so is {f (x_n)} in (Y, d_2). Is it true if f is only continuous? Justify. **CO 1**

11. State and prove Banach Fixed Point Theorem. **CO 2**

12. Prove that only connected subsets of $\mathbb{R}$ are intervals. **CO 2**

13. Suppose (X, d) is a metric space and $f: X \to X$ is an injective map. Set $D(x, y) = d(f(x), f(y))$. Prove that D is a metric on X. **CO 1**

Questions carrying 3 marks

1. Prove that a sequence is convergent in a metric space with discrete metric iff the sequence is eventually constant. **CO 1**

2. Show that the set of rational numbers is not complete in $\mathbb{R}$ with usual metric. **CO 1**

3. Prove that every convergent sequence is Cauchy but the converse is not true. **CO 1**

4. Prove that (a, b] is connected with usual metric of $\mathbb{R}$. **CO 1**

5. Prove that the space $\mathbb{Q}$ of rational numbers with subspace metric of the usual metric of $\mathbb{R}$ is not connected. **CO 2**

6. Prove that a metric space (X, d) having the property that every continuous map f : X → X has a fixed point, is connected. **CO 2**

7. Let (X, d) be a complete metric space and T : X → X be a contraction on X. Then for x ∈ X, show that the sequence $\{T^n x\}$ is a convergent sequence. **CO 1**

8. Prove that compact subsets of a metric space are closed and bounded. **CO 1**

9. Give an example to show that $(A \cup B)^o = A^o \cup B^o$ and $(A \cap B)' = A' \cap B'$ need not be true. **CO 1**
10. Suppose (X, d) is a metric space where d is the discrete metric on X and S is non empty subset of X. Then find the derived set S'. **CO 1**
11. Suppose (X, d) is a metric space where d is the discrete metric on X and S is non empty subset of X. Then find the interior S^o. **CO 1**
12. Define the usual metric, sup metric and taxi cab metric on $\mathbb{R}^2$. **CO 1**
13. If (X, d) is a metric space then construct a new metric on X using d. **CO 1**
14. Prove that any open ball is an open set. **CO 1**
15. Prove that if (X, d) is a metric space with discrete metric d then its every subset is an open set. **CO 1**
16. Prove that if (X, d) is a metric space with discrete metric d then its every subset is a closed set. **CO 1**

Multiple Choice Questions (2 marks)

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1. Let A be a subset of a metric space (X, d). Then $\{x \in X : d(x, A) = 0\}$ is : **CO 1**

a. equal to closure of A and is compact
b. equal to closure of A but not necessarily compact
c. equal to A
d. equal to $\{0\}$.

2. Which of the following set is complete in $\mathbb{R}$ with usual metric? **CO 2**

a. $\mathbb{Q}$
b. $\mathbb{R} - \mathbb{Q}$
c. $\mathbb{N}$
d. (0,1]

3. What is the diameter of $\mathbb{Q}$ in $\mathbb{R}$ with discrete metric? **CO 1**

a. Infinity
b. Not defined
c. 0
d. 1

4. Which of the following is not true about interior of a set? **CO 1**

a. It is always open
b. It is always closed
c. It is both open and closed
d. None of the above

5. Which of the following sets are not closed in $\mathbb{R}$ with usual metric? **CO 1**

a. $\{0,1,2,\dots,100\}$
b. $\mathbb{R}$
c. $\mathbb{Z}$
d. $\mathbb{Q}$

6. If the boundary of a set is empty then what can you conclude about the set? **CO 1**

a. The set is open
b. The set is not closed
c. The set is always finite
d. The set is closed

7. If the set is a closed set then which of the following is true? **CO 1**

a. It is countable
b. It is uncountable
c. It is complete
d. It is compact

8. Suppose (X, d) is a metric space where d is the discrete metric on X and S is non empty uncountable proper subset of X. Then what is the derived set S'. **CO 1**

a) S b) $X \backslash S$ c) $\emptyset$ d) X

9. If A and B are non empty subsets of a metric space (X, d), then which of the following statement is not always true? **CO 1**

a) $(A \cap B)^o = A^o \cap B^o$ b) $(A \cup B)^o = A^o \cup B^o$

c) $(A \cup B)' = A' \cup B'$ d) $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$

10. If d is the taxi cab metric on $\mathbb{R}^2$ then compute $d(x, y)$ where $x = (-2,3)$ and $y = (1, -2)$. **CO 1**

a) 5 b) 4 c) $\sqrt{34}$ d) 8

Answer:

1. b
2. c
3. d
4. b
5. d
6. d
7. c
8. c
9. b
10. d

CC13: METRIC SPACE & COMPLEX ANALYSIS

UNIT 2 (COMPLEX ANALYSIS)

Questions carrying 5 marks

1. State and prove Cauchy-Reimann equations for a differentiable function. **CO 3**

2. State and prove sufficient condition of differentiability of a complex valued function. **CO 3**

3. Let $f: \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = \frac{(\bar{z})^2}{z}$ $for\ z \neq 0$ and $f(0)=0$. Show that Cauchy Reimann are satisfied at $z=0$ but the derivative fails to exists at $z=0$. **CO 3**

4. If f is analytic function of $z = x + iy$, then show that $\frac{\partial f}{\partial \bar{z}} = 0$. **CO 3**

5. Check whether $f(z) = e^{\bar{z}}$ satisfies Cauchy-Reimann equations. Comment on the existence of $f'(z)$. **CO 3**
6. State the necessary condition for the differentiability of a complex valued function. **CO 3**
7. Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be a power series with radius of convergence $R>0$. Show that f is differentiable on $|z| < R$. Show that $f'(z) = \sum_{n=1}^{\infty} na_n z^{n-1}$and it has radius of convergence R. **CO 4**
8. a. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \frac{e^{in\pi}}{n}$. **CO 4**

   b. Let $f(z) = \bar{z}$ *and* γ is the semicircle from 1 to –1 passing through i. Evaluate $\int_{\gamma} f(z)dz$

9. a. Let z_1 and z_2 be the images in the complex plane of two diametrically opposite points on the Riemann sphere under stereographic projection. Then show that $z_1\overline{z_2} = -1$. **CO 3**

   b. Prove or disprove : The image of the circle | z | = r (r ≠ 1) under the transformation $w = z + \frac{1}{z}$z is an ellipse. **CO 4**

Questions carrying 3 marks

1. Let f be analytic on a region G. If f assumes only real values on G, then show that f is a constant. **CO 3**
2. Prove that hyperbolic sin is an entire function. **CO 3**
3. If $u = x^2 - y^2$, then find its conjugate harmonic function. **CO 3**
4. Describe the domain in a complex plane. Is $Im\, z \neq 2$ a domain? **CO 3**
5. Show that $f(z) = |z|^2$ is not differentiable for all $z \neq 0$. **CO 3**
6. Show that $f(z) = |z|^2$ is differentiable at z=0. **CO 3**
7. Evaluate $\int_{|z|=2} \frac{e^z+z^2}{z-1} dz$ **CO 4**

Multiple Choice Questions (2 marks)

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1. If $f = u + i\, v$ is an analytic function and $v = xy$ then u equals to **CO 3**

a. $x^2 - y^2$
b. $x^2 + y^2$
c. $(x^2 + y^2)/2$
d. $(x^2 - y^2)/2$

2. Which of the following function is not an entire function? **CO 3**

a. z^2
b. e^z
c. $\sin z$
d. log z

3. For which value of m, $2x - x^2 + my^2$ is harmonic? **CO 3**

a. 1
b. -1
c. 2
d. -2

4. If $f(z)$ is an analytic function whose real part is constant then $f(z)$ is **CO 3**

a. function of z
b. function of x only
c. function of y only
d. constant

5. If $u = 2x^2 - 2y^2 + 4xy$, then find its conjugate harmonic function. **CO 3**

a. $-2x^2 + 2y^2 + 4xy + \text{constant}$
b. $4y^2 - 4xy + \text{constant}$
c. $2x^2 - 2y^2 + xy + \text{constant}$
d. $-2x^2 + 2y^2 - 4xy + \text{constant}$

6. If f(z) is an analytic function whose modulus is constant, then f(z) is a **CO 3**

a. Function of z
b. Constant
c. Function whose only imaginary part is constant
d. Function whose only real part is constant

7. Consider the exponential function $f(z) = e^z$ where $z = x + iy$. Find $f'(z)$. **CO 4**

a) e^z b) $e^{\bar{z}}$ c) Do not exist d) ie^z

8. Which of the following set is a domain?

a) $|3z + 4| \leq 3$ b) $Im\, z \neq 0$ c) $Im\, z > 2$ d) $|z + 4| \geq 4$ **CO 3**

9. Let $T(z) = \frac{az+b}{cz+d}$ be a bilinear transformation. Then ∞ is a fixed point of T if and only if **CO 4**

a) a=0 b) b=0 c) c=0 d) d=0

10. Let $(z) = |z|^2;\ z \in \mathbb{C}$. Then f is **CO 4**

a) continuous everywhere but differentiable nowhere

b) differentiable only at z = 0

c) continuous nowhere

d) differentiable everywhere.

11. The radius of convergence of the power series $\sum(4+3i)^n z^n$ is **CO 4**

a) 5 b) 1/5 c) 4 d) ¼

12. Value of the integral $\int_C \sec z\, dz$, where C is the unit circle with centre at origin, is **CO 4**

a) 2 b) 0 c) 1 d) -5

Answer:

1. d
2. d
3. a
4. d
5. a
6. b
7. a
8. c

Question Bank

SEC-B: SCIENTIFIC COMPUTING WITH SAGEMATH

Questions carrying 5 marks

1. Without using inbuilt functions write a program in Sage to find gcd of 36 and 100. **CO 3**
2. Define a matrix in sage whose rows are (1, – 3, 4, 7), (3, 4, 7, 9), (3, 7, 0, 11), (1, 3, – 4, 8). Give sage code to find the row reduced echelon form of the matrix. **CO 4**
3. Without using inbuilt functions write a program in sage to determine the total number of primes less than x, print the list of such primes and get an output for x = 98. **CO 3**
4. Without using inbuilt function write a program in sage to determine factorial of 10. **CO 3**
5. Without using inbuilt functions write a program in Sage to determine in decimal approximation the arithmetic mean and geometric mean of a list of numbers and get an output for the list 20, 15, 24, 31, 45, 17. **CO 3**
6. Write the Sage codes (without using inbuilt functions) to find and print the median of the numbers 11, 23, – 34, 40, 50. **CO 3**
7. Write sage code to plot the function $f(x) = x - \frac{1}{x}$ in $[-1,1]$.Write also the equation of the asymptote (if it exists). **CO 2**
8. Write a program in sage to find the GCD of two numbers a, b using Euclidean Algorithm. **CO 3**
9. What are open source softwares? "Sage is a free and open source mathematical software." — Explain in brief. **CO 1**
10. What will be the output of the following sage commands? **CO 1**
    (i) 70/12
    (ii) 70/12.0
    (iii) 70//12
    (iv) 70%12
    (v) n(70/12)
11. Write sage code to do the following where $f(x) = x^4 - x$ **CO 2**
    (a) find $f''(x)$.
    (b) find $f'''(x)$.
    (c) draw the graph of $f(x)$ where – 5 < x < 5 with colour blue.
    (d) draw the graph of $f''(x)$ where – 5 < x < 5 with colour green.
    (e) draw the graph of $f'''(x)$ where – 5 < x < 5 with colour red.
12. Write sage code to define a function sorting that will sort a list in increasing order. **CO 3**
13. Draw and shade the region bounded above by $y = e^x$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$. **CO 2**
14. Draw the graph of the function $y = e^x$ in [-2,2] along with its tangent line at x=1 in different colors and linestyles. **CO 2**
15. Write a Sage program that counts the number of vowels in a given string variable. **CO 3**
16. Use a while loop to check whether a given number is prime. **CO 3**
17. Write a Sage program that outputs a conversion table from liters to gallons. **CO 1**

18. Create a list of 100 randomly generated points from the interval $[-100, 100]$ and then find the second smallest value from the list. **CO 3**
19. Which of the following variable names are illegal? **CO1**

    number_of_steps
    dayOfWeek
    June 2016
    2DPlot
    function
    month#3
    Junne2016
20. Write sage commands to solve the following Initial Value Problem and plot the solution : **CO 4**

$$\frac{dy}{dx} = 1 - y, y(10) = 2$$

Questions carrying 3 marks

1) Write a program in sage to find the greatest of three given integers a, b, c. **CO 3**
2) Is the variable name index same as InDeX? Why or why not? **CO 1**
3) Given an amount of x total cents, write Sage code that outputs the same value in dollars and cents. For $x = 270$ your code should output "2 dollars, and 70 cents." **CO 1**
4) Write a Sage code that computes the sum of the squares of all prime numbers less than 2015. **CO 3**
5) Suppose today is Tuesday. Given a positive integer no_of_days, output the name of the day of the week that is exactly no_of_days days after Tuesday. **CO 1**
6) Write a Sage code that computes the smallest value from a randomly generated list of 100 numbers. **CO 3**
7) Define a function is_even() that returns true if a given value is even, false otherwise. **CO 3**
8) Define a function is_palindrome() that returns true if a given word is palindrome, false otherwise. **CO 3**
9) Write sage commands to solve the following Initial Value Problem and plot the solution : **CO 4**

$$y''' - y' = 1 - y, y(0) = 2, y'(0) = 4, y''(0) = -2$$

10) Write sage code for the following : Find the number of digits in 2021! and compute the number of zeros and the number of ones present in 2021!. **CO 3**
11) How many days are there in 10,000 hours? If right now is 7:01 am, what time will it be 10,000 hours later? **CO 1**
12) Write a Sage program that checks whether or not a given number is cube free. **CO 3**
13) Find the number of digits of the following number: $2^{2^{100}}$. **CO 1**
14) Create a list with 100 randomly generated points with both coordinates in the interval [20, 70] and plot them. **CO 1**
15) Define a function upper() that prints a given text using only uppercase characters. **CO 3**
16) Write a function letter_grade() that returns the letter grade of a given score. For example, letter_grade(98) should return A, and letter_grade(59) should return F. **CO 3**

17) Assuming the non-trivial solution exists, write sage commands to solve the following system of equations with matrices : **CO 4**

$$3x - 4y + 5z = 14$$
$$x + y - 8z = -5$$
$$2x + y + z = 7.$$

18) Write sage commands to solve the following Initial Value Problem and plot the solution : **CO 4**

$$y'' - y' = 1 - y, y(0) = 2, y'(0) = 4$$

19) Write a program in sage to find the sum of the following series for any finite n : **CO 1**

1.1 + 4.5 + 6.7 + … … … … … + 2n(2n + 1).

20) Consider the following program : **CO 3**

```
i=0
while i < 5:
    print(i)
    i=i+1
    if i= =3:
        break
    else:
        print(0)
```

What will be the output of the program segment?

Multiple Choice Questions (2 marks)

1. What will be your code in Sage, if you want all the square roots of 4? **CO 1**
(i) sqrt(4) (ii) sqrt(4)
(iii) sqrt(4, all = = true) (iv) sqrt(4, all=true).

2. Which of the following expression is not a Boolean expression? **CO 1**

a) $!(2 \geq 3) or (3 < 4)$
b) $(2 = 3) or (3 < 4)$
c) $\left((2 + 3) > 5\right) and (3 < 4)$
d) $(4 > 5) and (3 < 4)$

3. Which of the following variable name is legal in Sage Math? **CO 1**
a) $while$ b) $Step23\_d$ c) $2for$ d) $step\#$

4. If $L = [2,4,6,9,13,12,14]$, then what is $L[3] + L[5] + len(L)$? **CO 1**
a) 28 b) 25 c) 27 d) 24

5. What is the output of following sage code **CO 3**

```
X=3
for i in range(4):
    X=X+2
print(X)
```

a) 13 b) 11 c) 9 d) 15

6. What is the output of the following sage code? **CO 3**

```
X=3
A=2
while(X<16):
    X=X+A
    A=A+3
print(X)
```

a) 18 b) 20 c) 16 d) 15

7. Consider the following for loop command in Sage: **CO 3**

```
for i in range(20):
    block_statements
```

How many times the block of statements will be evaluated in the above sage code.

a) 21 b) 20 c) 19 d) 0

8. Consider the following sage code. **CO 3**

```
S=2
for i in [2,4,..,8]:
    S=S+1
print(S)
```

What is the output of the above Sage code?

a) 5 b) 6 c) 9 d) 12

9. What will be the output of the following sage code? **CO 1**

```
a=4
b=2
```

print(a+b*2)
(i) 36 (ii) 10
(iii) 8 (iv) 12

10. What will be your code in Sage to find the natural logarithm of 100 in decimal approximation? **CO 1**
(i) N(log(100)) (ii) n(log(100,10))
(iii) n(logexp(100)) (iv) N(log e(100))

11. What is the correct code in Sage to evaluate the value of e correct up to 100 digits? **CO 1**
(i) n(e, 100) (ii) n(e, digits=100)
(iii) N(e, prec=100) (iv) N(e,100).

12. The output of the Sage code : 4*(10//4) + 10%4 == 10, 3*3<3 is **CO 1**
(i) (True, False) (ii) (True, True)
(iii) (False, True) (iv) (False, False).

13. What will be the output of the following sage code? **CO 3**
a =[1, 3]
b=[10, 20, a]
print(b)
(i) [10, 20, [1,3]]

(ii) [10, 20, 1, 3]
(iii) An error will occur (iv) [1, 3, 10, 20]

14. Which of the following is NOT the correct option for a^{b^c} in sage? **CO 1**
(i) a ^ b ^ c (ii) a * *b * *c
(iii) a ^^ b ^^ c (iv) a ^ b * *c.

15. What will be the result of the following Sage code: exp(11*8+5) – e^(11*8+5)? **CO 1**
(i) 0 (ii) error: “exp” is not defined
(iii) error: “e” is not defined (iv) error: “e^” is not defined.

16. What will be the result of the following Sage code: 2**3-4+5*60%4//2? **CO 1**
(i) 0 (ii) 4
(iii) 2 (iv) 10

17. What is the correct code in Sage to plot the function $x^4 - x$ in $-2 < x < 2$? **CO 2**
(i) plot(x^4 – x, –2, 2) (ii) plot($x^4 - x$,-2,2)

(iii) plot($x^4 - x$,range[- 2, 2]) (iv) plot($x^4 - x;\ -2,2$)

18. . What will be the result of the following Sage code: n(1.234547865, digits=5)? **CO 1**
(i) 1.23454 (ii) 1.2344
(iii) 1.23455 (iv) 1.234

19. What is the output of the following Sage Code? **CO 3**

```
S1="cat"
S2="dog"
S3=" "
for i in [0,1]:
    S3=S1+S3+S2
print(S3)
```

(i) cat dog (ii) catcat dogdog

(iii)catdog catdog (iv) dogdog catcat

20. Which of the following command is used to find the determinat of a matrix. **CO 4**
(i) determ() (ii) det() (iii) | | (iv) det_()

Answers:

1. iv 2. b 3. c 4. a 5. b 6. a 7. b 8. b 9. iii 10. i 11. ii 12. i 13. i
14. iii 15. i 16. ii 17. i 18. i 19. ii 20. Ii

Question Bank

DSE-B1: LINEAR PROGRAMMING & GAME THEORY

Questions carrying 5 marks

1) Determine all basic feasible solution of the set of equations $2x_1 + 6x_2 + 2x_3 + x_4 = 3$, $6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$, also identify basic & non basic variables in each case . **CO 1**

2) Show that the L.P.P Max z = $x_1 + 4x_2 + 3x_3$ **CO1, CO 2**
Subject to : $2x_1 - x_2 + 5x_3 = 40$, $x_1 + 2x_2 - 3x_3 \geq 22$, $3x_1 + x_2 + 2x_3 = 30$
x_1 , x_2 , $x_3 \geq 0$ has no feasible solution .

3) If the k-th primal constraint is an equation , then the corresponding dual variable w_k is unrestricted in sign . Verify this in Min z = x+y+z **CO 3**
Subject to : x-3y+4z = 5 , x-2y ≤ 3 , 2y-z≥ 4 , $x , y \geq 0$ z is unrestricted .

4) Solve the transportation problem : **CO 3**

	Stores						Capacities
Warehouse	1	2	3	4	5	6	
1	9	12	9	6	9	10	5
2	7	3	7	7	5	5	6
3	6	5	9	11	3	11	2
4	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	

5) Prove that for a 2× 2 Game with mixed strategies with the pay off matrix **CO 4**
$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ solution always exists if $(a_{11} + a_{12}) \neq (a_{12} + a_{21})$ & value of the game
$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} + a_{21}}$

6. Prove that every extreme point of the convex set of all feasible solutions of the system of equations **CO 1**

$$AX = b, \quad X \geq 0$$

corresponds to a basic feasible solution.

7. Solve the LPP by simplex method: **CO 3**

Maximize $z = 4x_1 + 3x_2 + 4x_3 - 5x_4$

Subject to
$$3x_1 + x_2 \le 15$$
$$3\,x_1 + 4x_2 \le 24$$
$$x_1, x_2 \ge 0$$

8. Find the dual of the following LPP: **CO 4**

Maximize $z = x_1 - x_2 + 4x_3 - 5x_4$

Subject to
$$x_1 + 2x_2 \ge -4$$
$$x_1 + 2x_2 - x_3 = 5$$
$$x_1 + 5x_3 - 3x_4 \le -3$$
$x_1, x_2 \ge 0$, and x_3, x_4 are unrestricted in sign

9. Define a two-person zero-sum game? Evaluate the saddle point and value of the following game **CO 4**

$$\begin{array}{cc} & B \\ & \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ A \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} 4 & 2 & 2 & 5 \\ -2 & -1 & 4 & -3 \\ 6 & 2 & 3 & 3 \\ 4 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

10. State and prove fundamental theorem of duality. **CO 4**

11. A pharmaceutical firm produces two products A and B. Each unit of product A requires 3 hrs. of operation–I and 4 hrs. of operation-II, while each unit of product B requires 4 hrs. of operation-I and 5 hrs. of operation-II. Total time available for operation I and II are 20 hrs. and 26 hrs. respectively. Product A sells at a profit of Rs. 10 per unit, while product B sells at a profit of Rs. 20 per unit. Formulate the problem as an LPP to maximize the profit. **CO 1**

12. Suppose that a constraint in a given L.P.P. (considered to be primal) is an equality. Prove that the corresponding dual variable is unrestricted in sign. **CO 4**

13. Prove that the number of basic variables in a transportation problem is at most $(m + n - 1)$, where 'm' is the number of origins and 'n' the number of destinations. **CO 2**

14. Solve the following Travelling Salesman Problem so as to minimize the cost per cycle **CO 2**

$$\begin{array}{cc} & \text{To} \\ From & \begin{bmatrix} \infty & 10 & 25 & 25 & 10 \\ 1 & \infty & 10 & 15 & 2 \\ 8 & 9 & \infty & 20 & 10 \\ 14 & 10 & 24 & \infty & 15 \\ 10 & 8 & 25 & 27 & \infty \end{bmatrix} \end{array}$$

15. Reduce the following pay-off matrix to a 2 × 2 matrix by dominance property and then solve the game problem, where A is the maximizing player and B is the minimizing player : **CO 4**

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 2 & 1 & -2 & -3 \\ 4 & 3 & 4 & -2 & 0 \\ 5 & 1 & 2 & 5 & 6 \\ 1 & 2 & 1 & -3 & -3 \end{bmatrix} \end{array}$$

16. Prove that the objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. **CO 1**

17. State the mathematical formulation of a general transportation problem. Also show that an assignment problem is a special case of transportation problem. **CO 2**

Questions carrying 3 marks

1) Convert the problem to its dual form : Min z = $x_1 + x_2 + x_3$ **CO 4**
   Subject to : $x_2 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \le 3$, $2x_2 - x_3 \ge 4$
   x_1 , x_2 , $x_3 \ge 0$.

2) Prove that the set S = $\{(x_1, x_2)/{x_1}^2 + {x_2}^2 = 4\}$ is not a convex set . **CO 1**

3) What is a minimax principle ? Explain it with an example of 3×3 size pay off matrix . **CO 4**

4) Prove that the vectors (1,1,1) ,(1,1,0) , (1,0,0) form a basis of E^3, form a new basis from the original basis solution . **CO 1**
5) State the mathematical formulation of a general transportation problem. **CO 2**
6) Show that an assignment problem is a special case of transportation problem. **CO 2**
7) Prove that the set of all feasible solutions of a linear programming problem is a convex set. **CO 1**
8) Examine whether the set $S = \{(a,b): ab \ge 0, a \ge 0, b \ge 0\}$ is convex or not. **CO 1**
9) Mention two situations of degeneracy that may occur in solving an L.P.P. **CO 3**
10) Define a convex set. Also define a convex polyhedron and give example of a convex set which is not a convex polyhedron. **CO 1**
11) Write the standard form of L.P.P. corresponding to the following problem of game from the point of view of player B : **CO 4**

$$\begin{array}{cc} & B \\ A & \begin{bmatrix}1 & -1 & 3\\ 3 & 5 & -3\\ 6 & 2 & -2\end{bmatrix}\end{array}$$

Multiple Choice Questions (2 marks)

1) The feasible region of solution of a LPP involving three variables is

a. A hyperplane **CO 1**
b. A polyhedron
c. A convex hull
d. A convex set

2) If the dual problem has no feasible solution and the primal problem has a feasible solution , then the primal objective function is **CO 4**

a. Bonded
b. Unbounded
c. Noting can be estimated
d. None of these

3) In a transportation problem involves m origins and n destinations , the number of basic variables is **CO 2**

a) At least (m+n-1)
b) At most (m+n-1)
c) At least (m+n)
d) At most (m+n)

4) Which of the following is not true in the simplex method ? **CO 2, CO 3**

a) At each iteration , the objective value either stays the same or improves
b) It indicates an unbounded or infeasible solution
c) It signals optimality
d) It converges in at most m steps where m is the number of constraints

5. Suppose that the objective function of an L.P.P. assumes its optimal value at more than one extreme point. Then **CO 1**

(i) the convex combination of these extreme points will improve the value of the objective function.

(ii) the value of the objective function will be different for different convex combinations of these extreme points.

(iii) it indicates that the number of basic feasible solutions is degenerate.

(iv) every convex combination of these extreme points also gives the optimal value of the objective function.

6. Consider two sets

$$X = \{(a, b): a + b \le 5, 4a + 4b \ge 16\}$$

CO 1

$$Y = \{(a, b): a + b \le 5, 4a + 4b \ge 4\}$$

Then

(i) X is a convex set, but Y is not a convex set.

(ii) X is not a convex set, but Y is a convex set.

(iii) both X and Y are convex sets.

(iv) none of X and Y are convex sets.

7. The optimal mixed strategies for the players A, B and the value of the game (v) with pay-off matrix

CO 3

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \end{array}$$

will be

(i) $\left(\frac{5}{6}, \frac{1}{6}\right), \left(\frac{2}{3}, \frac{1}{3}\right), v = \frac{7}{3}$

(ii) $\left(\frac{4}{6}, \frac{2}{6}\right), \left(\frac{2}{3}, \frac{1}{3}\right), v = \frac{7}{3}$

(iii) $\left(\frac{5}{6}, \frac{1}{6}\right), \left(\frac{2}{3}, \frac{1}{3}\right), v = \frac{5}{3}$

(iv) $\left(\frac{3}{6}, \frac{3}{6}\right), \left(\frac{2}{3}, \frac{1}{3}\right), v = \frac{7}{3}$

8. Given the following cost matrix of a transportation problem

CO 2

$$\begin{array}{ccc} & \text{Destinations} & \\ Origins & \begin{bmatrix} 4 & 6 & 9 & 5 \\ 2 & 6 & 4 & 1 \\ 5 & 7 & 2 & 9 \end{bmatrix} & \begin{array}{c} 16 \\ 12 \\ 15 \end{array} \\ & \begin{array}{cccc} 12 & 14 & 9 & 8 \end{array} & \end{array}$$

the cost of transportation according to North-West corner rule is given by

(i) 220 (ii) 226 (iii) 250 (iv) 215

9. The maximum number of basic solutions for an $m \times n$ LPP is

CO 2

(i) n (ii) m (iii) ${}^{n}C_{m}$ (iv) $m + n - 1$.

10. Consider a game of size $m \times n$ with pay-off matrix $A = \left(a_{ij}\right)_{m \times n}$. If a fixed number be added to each element of A, then **CO 3, CO 4**
(i) the optimal strategies remain unchanged.
(ii) the value of the game remains unchanged.
(iii) the value of the game is decreased by that number.
(iv) both the optimal strategies and the value of the game remain unchanged.

11. Given the system of constraints : **CO1**

$$a + 2b + 3c + 4d = 7$$
$$2a + b + c + 2d = 3$$

(i) (0, 2, 0, 1) is a basic solution,
(ii) (1, 1, 0, 0) is a basic solution,
(iii) (0, 2, 1, 0) is a basic solution,
(iv) (2, 0, 1, 4) is a basic solution

12. For the following cost matrix **CO 2**

$$\begin{array}{cc} & \text{Machine} \\ Job & \begin{bmatrix} 4 & 4 & 6 \\ 10 & 7 & 11 \\ 4 & 5 & 12 \end{bmatrix} \end{array}$$

the minimum cost of assignment is
(i) 15 units
(ii) 17 units
(iii) 20 units
(iv) 22 units

13. The assignment problem will have alternate solutions when **CO 2**
(i) total opportunity cost matrix has at least one zero in each row and column.
(ii) the total opportunity cost matrix has at least two zeros in each row and column.
(iii) there is a tie between zero opportunity cost cells.
(iv) two diagonal elements are zeros.

14. Consider the following pay-off matrix of a game. Identify the dominance in it. **CO 3, CO 4**

$$\begin{array}{cc} & \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{array}{c} P \\ Q \\ R \end{array} & \begin{bmatrix} 4 & 4 & 6 \\ 10 & 7 & 11 \\ 4 & 5 & 12 \end{bmatrix} \end{array}$$

(i) P dominates Q
(ii) Y dominates Z
(iii) Q dominates R
(iv) Z dominates Y.

15. Consider the game with the pay off matrix : **CO3, CO 4**

$$\begin{array}{cc} & \text{Player B} \\ Player\ A & \begin{bmatrix} p & 7 & 3 \\ -2 & p & 8 \\ -3 & 4 & p \end{bmatrix} \end{array}$$

The value of p for which the game is strictly determinable satisfies
(i) $-8 \leq p \leq -3$
(ii) $-3 \leq p \leq -2$
(iii) $-2 \leq p \leq 3$
(iv) $-8 \leq p \leq 7$.

16. The number of extreme points of the convex set S = {(x, y) : | x | ≤ 1, | y | ≤ 1} is
(i) 0 (ii) 2 **CO 1**
(iii) 4 (iv) infinitely many.

17. Let $x = \{(a, b): a^2 + b^2 = 1\}$ and y is the set of all convex combinations of the vertices of a cube. Then
(i) x is a convex polyhedron, but y is not. **CO 1**
(ii) x is not a convex polyhedron, but y is a convex polyhedron.
(iii) both x and y are convex polyhedrons.
(iv) neither x nor y is a convex polyhedron.

18. Consider an L.P.P
Maximize z = cx,
subject to the constraints **CO 1, CO 2**
Ax = b, x ≥ 0
(The symbols have their usual meaning).
Then the problem admits of an unbounded solution, if at any iteration of the simplex algorithm,
(i) at least one index number is found to be negative and all elements in the column corresponding to that negative index are non-positive.
(ii) at least one index number is found to be negative and all elements in the column corresponding to that negative index are all positive.
(iii) at least one index number is found to be positive and all elements in the column corresponding to that positive index are non-positive.
(iv) at least one index number is found to be positive and all elements in the column corresponding to that positive index are positive.

19. A degenerate BFS in a balanced TP with m origins and n destinations will consist of
(i) at least (m + n – 1) positive-variables
(ii) at most mn – (m + n – 1) positive variables **CO 2**
(iii) at most m + n – 1 positive variables
(iv) at most m + n – 2 positive variables.

20. z = 20a + 9b
Subject to 2a+ 2b ≥ 36

$6a + b \geq 60$
$a \geq 0, b \geq 0$
The minimum value of z is
(i) 360 at (18, 0) (ii) 336 at (6, 4)
(iii) 540 at (0, 60) (iv) 0 at (0, 0).

Question Bank on Advanced Mechanics

MTMA Sem-VI, Paper-DSE-A

MCQ (Each Question carries 2 marks)

(Each question below is followed by four possible answers of which exactly one is correct. Choose the correct answer with proper justification.)

1. The poisson bracket $\{|\vec{r}|, |\vec{p}|\}$ has the value
   a) $|\vec{r}||\vec{p}|$ b) $\hat{r}.\hat{p}$ c) 3 d) 1. <CO-2>
2. Let H(q,p) and L(q,$\dot{q}$) denote respectively the Hamiltonian and Lagrangian of a autonomous system with p as generalized momentum and q as generalized coordinate vector then
   a) H remains conserved in the motion.
   b) H is simply the total energy of the system. <CO-1>
   c) p is conserved if H is independent of q
   d) p is conserved if L is independent of q
3. Let H and L be the Hamiltonian and Lagrangian respectively of a free particle of mass m and velocity v ,then
   a) H and L are independent of each other
   b) H and L are related but h dependent of v <CO-1>
   c) H and L are equal
   d) Both H and L are quadratic in v.
4. Which of the following is/are canonical transformation
   a) Q = log((1/q) sin(p)), P = q.cot(p)
   b) P = 2(1+$\sqrt{q}$.cos(p)), Q = log(1+$\sqrt{q}$.cos(p)) <CO-3>
   c) P = $\frac{1}{2}(p^2 + q^2)$,Q = $tan^{-1}(\frac{q}{p})$
   d) Q = q.tan(p), P = log(sin(p))
5. Consider a mass m moving in an square central force with characteristic coefficient μ and described by the Lagrangian L$(r, \dot{r}, \theta, \dot{\theta}) = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{\mu m}{r^2}$, then
   a) The generalized momentum of the system $p_r = m\dot{r},\ p_\theta = mr^2\dot{\theta}$
   b) H $= \frac{1}{2m}(p_r{}^2 + \frac{p_\theta{}^2}{r^2}) - \frac{\mu m}{2r}$ <CO-2>
   c) H $= \frac{1}{2m}(p_r{}^2 + \frac{p_\theta{}^2}{r^2}) - \frac{\mu m}{r^2}$
   d) $p_r = m\dot{r},\ p_\theta = -mr^2\dot{\theta}$
6. A bead slides on a smooth rod which is rotating about one end in a vertical plane with uniform angular velocity ω. The equation of motion and L are <CO-3>

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a) $\ddot{r} + gsin(\theta) = r\omega^2$

b) $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - mgrsin\theta$

c) $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + mgrsin\theta$

d) $\ddot{r} + gsin\theta = 0$

7. The Lagrange system of equation in terms of polar coordinates (r, θ) is given by $L = \frac{m}{2}\dot{r}^2 + \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - mgr(1 - cos\theta)$,then the equation of motion is

a) $\ddot{r} = \frac{1}{2}(r\dot{\theta}^2 - g(1 - cos\theta))$

b) $\ddot{\theta} = \frac{g}{r}\cos\theta$ <Co-2>

c) $\ddot{\theta} = \frac{g}{r}\sin\theta$

d) $\ddot{r} = \frac{1}{2}(r\dot{\theta}^2 + g(1 - cos\theta))$

8. For Hamilton $H=\frac{1}{2}\left[\frac{1}{q^2} + p^2q^4\right]$, then the value of L is

a) $L=\frac{\dot{q}^2}{2q^4} + \frac{1}{2q^2}$

b) $L=\frac{\dot{q}^2}{2q^4} - \frac{1}{2q^2}$ <Co-2>

c) $L=\frac{\dot{q}^2}{2q^3} - \frac{1}{2q^2}$

d) $L=\frac{\dot{q}^2}{2q^3} + \frac{1}{2q^2}$

9. Consider a particle of mass m in simple oscillation about the origin with spring constant k, then the Lagrangian L is

a) $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2, p = m\dot{x}^2$ <Co-2>

b) $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2, p = m\dot{x}$

c) $L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2, p = m\dot{x}$

d) None of the above

10. A particle of mass m moves in a plane. Lagrange's equation of motion and Kinetic energy in polar coordinates are

a) $T=\frac{1}{2}m(\dot{x}^2 + \dot{y}^2), 2m\dot{r}\dot{\theta} + rm\ddot{\theta} = F_\theta$

b) $T=\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2), 2m\dot{r}\dot{\theta} + rm\ddot{\theta} = F_\theta$ <Co-3>

c) $T=\frac{1}{2}m(\dot{r}^2 + r\dot{\theta}^2), 2m\dot{r}\dot{\theta} - rm\ddot{\theta} = F_\theta$

d) $T=\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2), 2m\dot{r}\dot{\theta} + rm\ddot{\theta} = 0$

11. Which of the following is/are true

a) $L = T - V$

b) $H = T + V$

c) Generalized momentum corresponding to a cyclic coordinate is constant <Co-2>

d) None

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12. The Hamiltonian corresponding to the Lagrangian L= $a\dot{x}^2 + b\dot{y}^2 - kxy$ is

a) $\frac{p_x{}^2}{2a}+\frac{p_y{}^2}{2b}$+kxy

b) $\frac{p_x{}^2}{4a}+\frac{p_y{}^2}{4b}$ −kxy <Co-3>

c) $\frac{p_x{}^2}{4a}+\frac{p_y{}^2}{4b}$+kxy

d) $\frac{p_x{}^2}{4ab}+\frac{p_y{}^2}{4ab}$+kxy

13. Hamiltonian canonical equations of motion for a conservation system is/are

a) $\frac{dq_i}{dt} = -\frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$

b) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

c) $\frac{dq_i}{dt} = -\frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$ <Co-2>

d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$

14. The generalized coordinate q_k *is* called cyclic if <Co-2>

a) $\dot{q}_k = 0$,b) $p_k = 0$,c) $\frac{\partial L}{\partial q_k} = 0$, d) $\frac{\partial L}{\partial q_k} = 1$

15. If q_k is a cyclic coordinate then <Co-1>

a) $\dot{q}_k = 0$, b) $\dot{p}_k = constant$, c) $\frac{\partial L}{\partial \dot{q}_k} = 0$, d) none

16. Lagrangian of the Sun-Earth system is

a) $L = \frac{1}{2}mr^2\dot{\theta}^2 + \frac{GMm}{r}$

b) L= $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GMm}{r}$, <Co-2>

c) $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GMm}{r}$,

d) L= $\frac{1}{2}m\dot{r}^2 - \frac{GMm}{r}$,

17. The order of Lagrangian equation of motion is <Co-1>

a) 1, b) 2, c) 3,d) 4

18. The Lagrange's equation of motion for a system is equivalent to <Co-1>

a) Newton's Equation of motion

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b) Laplace's Equation of motion

c) Poisson's Equation of motion

d) none

19. If the Lagrangian does not depend on time explicitly,then

a) Hamiltonian is constant of motion <Co-2>

b) Hamiltonian does not constant of motion

c) K.E. is constant

d) P.E. is constant

20. Poisson bracket of two integral of motion is <Co-2>

a) zero b) unity c) integral of motion d) infinite

21. which of the following transformation is canonical?

a) $P = q, Q = p$,b) $P = q, Q = -p$,c) $P = -q, Q = -p$,c) $P = -q, Q = p$, <Co-2>

22. If the transformation $Q = q^{\alpha} sinp\beta, P = q^{\alpha} cosp\beta$ is canonical then the values of α, β are

a) $\alpha = 1, \beta = 0$ b) $\alpha = \frac{1}{2}, \beta = 2$ c) $\alpha = \frac{1}{2}, \beta = \frac{1}{2}$ d) $\alpha = \frac{1}{2}, \beta = -2$ <Co-2>

23. Two particles are connected by a rod of length $l = f(t)$. The nature of the constraint is

a) scleronomic and holonomic b) rheonomic and non-holonomic c) scleronomic and non-holonomic d) rheonomic and holonomic. <Co-2>

24. Consider a planet of mass$m$ orbiting around the Sun under the inverse square law of attraction $\frac{\mu m}{r^2}, \mu > 0$. If the position of the planet at time t is given by the polar coordinates (r, θ) , then the Lagrangian L of the system is given by <Co-2>

a) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{\mu m}{r}$ b) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{\mu m}{r}$ c) $\frac{1}{2}m(\dot{r}^2 + \dot{\theta}^2) + \frac{\mu m}{r}$ d) $\frac{1}{2}m(\dot{r}^2 + \dot{\theta}^2) - \frac{\mu m}{r}$

25. The homogeneity of time leads to the law of conservation of <Co-1>

a) linear momentum b) angular momentum c) energy d) parity

26. The Lagrangian of a system is given by$L = \frac{1}{2}ml^2(\theta + sin\theta.\dot{\varphi^2}) - mglcos\theta$ where m, l, g are constants. Which of the following is conserved. <Co-2>

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a) $\dot{\varphi} sin\theta$ b) $\dot{\varphi}/sin\theta$ c) $\dot{\varphi} sin^2\theta$ d) $\dot{\varphi}/sin^2\theta$.

27. A Particle moves in two dimension with potential $V(x,y) = x + 2y$. which of the following is a constant of motion?

a) $p_y - 2p_x$ b) $p_x - 2p_y$ c) $p_x + 2p_y$ d) $p_y + 2p_x$ <CO-2>

28. The Hamiltonian of a system is given by $H(q_1, q_2 ; p_1, p_2) = K{p_1}^2 + \frac{K}{{q_1}^2}{p_2}^2 + \frac{l}{q_1}$, where q_1, q_2 are generalized coordinates and p_1, p_2 are the generalized momenta and K, l are constants ,then <CO-2>

a) $p_1 = Kt + l$ b) $p_2 = Kt + l$ c) p_1 is independent of time. d) p_2 is independent of time.

29. If in a scleronomic system the kinetic energy be a homogeneous function of velocity, then $\sum \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} - L$ will be equal to <CO-2>

a) $T + V$ b) T c) $T - V$ d) V, the symbols have their usual meanings.

30. A linear transformation of a generalized coordinate q and corresponding momenta p, to Q and P given by $Q = q + p, P = q + \alpha p$ is canonical if the value of the constant α is

a) -1 b) 0 c) 1 d) 2. <CO-3>

31. The Poisson bracket $[x, xp_y + yp_x]$ is equal to <CO-2>

a) $-x$ b) y c) $2p_x$ d) p_y.

32. Two particles are connected by a rigid weightless rod of constant length. The degrees of freedom of this two particle system is <CO-2>

a) 3 b) 4 c) 5 d) 6.

Long Questions: (Each Question carries 3/5 marks)

1. What is canonical Transformation? Show that the transformation $P = \frac{1}{2}(p^2 + q^2), Q = tan^{-1}(\frac{q}{p})$ is canonical. <CO-3>

2. If X, Y, Z are three dynamical variables then prove that $[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$. <CO-2>

3. What is Hamilton's Principle? Deduce Hamilton's Principle from D'Alembert's Principle. <CO-1>
4. Deduce Lagrange's Equation of 2nd kind for a conservative holonomic dynamical system. <CO-3>
5. Establish Equation of motion for a Simple pendulum. <CO-3>

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6. Establish necessary and sufficient condition for canonical transformation. <Co-2>
7. Establish Hamilton's equations of motion in terms of Poisson bracket. <Co-2>
8. Derive Lagrange's equation of motion from variational method. <Co-3>
9. Derive expression for kinetic energy and hence show that if the transformation equation does not contain time explicitly , kinetic energy is a homogeneous quadratic expression for generalized velocities. <Co-3>
10. Derive the Lagrange's equation of motion for a pendulum in spherical polar coordinates of length l. <Co-3>
11. For a dynamical system $T=\frac{1}{2}\{(1+2k)\dot{\theta}^2+2\dot{\varphi}\dot{\theta}+\dot{\varphi}^2\}$, $V=\frac{n^2}{2}\{(1+k)\theta^2+\varphi^2\}$ where θ *and* φ are coordinates , n, k are positive constants. Write down the Lagrange's equations of motion and deduce that <Co-3>

$$\ddot{\theta}-\ddot{\varphi}+n^2\left(\frac{1+k}{k}\right)(\theta-\varphi)=0$$

and if $\theta=\varphi, \dot{\theta}=\dot{\varphi}$ at $t=0$ *then* $\theta=\varphi$ for all t.
12. In a dynamical system of two degrees of freedom,the kinetic energy $T=\frac{1}{2}\frac{\dot{q_1}^2}{a+bq_2^2}+\frac{1}{2}\dot{q_2}^2$ and potential energy $V=c+dq_2^2$. Find q_1 *and* q_2 where a, b, c, d are constants. <Co-3>
13. The Hamiltonian of a dynamical system is given by $=qp^2-qp+bp$, where $a$is a constant. Solve the problem. <Co-3>
14. If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, Show by direct substitution that
$L'=L+\frac{dF(q_1,q_2,q_3,\ldots\ldots,q_n)}{dt}$ also satisfies Lagrange's equations where F is any arbitrary,but differentiable function of its arguments. <Co-3>
15. Deduce Lagrange's equation of motion from Hamilton's principle. <Co-3>
16. State Noether's theorem. What do you mean by homogeneity of space? Show that homogeneity of space leads to the conservation of linear momentum. <Co-2,3>
17. i) What do you mean by Δ-variation of the path of a system? State principal of least action. <Co-2>

    ii) For a simple pendulum, obtain the expression of Hamiltonian function and derive Hamilton's canonical equations of motion. <Co-3>
18. The Hamiltonian of a dynamical system is given by

$$H=\frac{1}{2}\sum_{i=1}^{3}(p_i^2+\mu^2 q_i^2),$$

where q_i , p_i are the generalized coordinates and momenta and μ is a constant. Show that <Co-3>

$F=q_2p_3-q_3p_2$ is a constant of motion.
19. What is an action of an mechanical system? State the principal of stationary action and hence show how does it lead to Format's principal. <Co-3>
20. Show that in phase space area remains conserved under Hamiltonian flows.

<Co-3>

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21. If u and v are two constants of motion in a given holonomic system, prove that the Poisson bracket $[u , v]$ is also a constant of motion. <CO-3>
22. Use Hamilton-Jacobi technique to solve the one-dimensional harmonic oscillator problem. <CO-3>
23. Show that the transformation $Q = qcos\theta - \frac{p}{\mu\omega} sin\theta, P = \mu\omega q sin\theta - pcos\theta$ is a canonical transformation for all values of θ. Also find $[Q, P]_{(q,p)}$. <CO-3>
24. i) Show that the transformation $= qtanp$, $P = \log(\sin p)$ is canonical.
ii) Find the canonical transformation defined by the generating function $F_1(q, Q) = qQ - \frac{1}{2} m\omega q^2 - Q^2/(4m\omega)$, m, ω are constants. <CO-3>
25. A particle of mass m and coordinate q has the Lagrangian $L = \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$, where λ is a constant. Find the Hamiltonian and deduce Hamilton's equation's equation of motion. <CO-3>
26. State and prove the Principle's of least action. Obtain the modified Hamilton's principle from Principle's of stationary action. <CO-1,3>
27. Derive the Hamilton-Jacobi equation for Hamilton's principal function S. Solve the Hamilton-Jacobi equation for the system whose Hamiltonian is given by $H = \frac{p^2}{2} - \frac{\mu}{q}$. <CO-3>
28. Define Poisson bracket. If $F(p, q, t)$ and $G(p, q, t)$ are two constants of motion the show that the Poisson bracket $[F , G]$ is also a constant of motion . <CO-1,3>

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Question Bank on Bio-Mathematics

MTMA **SEM-V** **Paper-DSEA-1**

MCQ (Each Question carries 2 marks)

(Each question below is followed by four possible answers of which exactly one is correct. Choose the correct answer with proper justification.)

1. In the following growth model

   $\frac{dN}{dt} = rN(1 - \frac{N}{K})$, K being the carrying capacity, r(>0) being the growth rate,if N_0 be the initial population size, then the population doubling time exists if <CO-2>

   i) $K < 2N_0$ ii) $K = 2N_0 1$ iii) $K > 2N_0$ iv) $K = N_0$

2. In the theta-logistic growth model

   $\frac{1}{N}\frac{dN}{dt} = r[1 - (\frac{N}{K})^{\theta}]$, $\theta(> 0)$ being a parameter, K being the carrying capacity, the equilibrium point $N = K$ is <CO-2>

   i) stable ii) stable but not asymptotically stable iii) asymptotically stable iv) unstable

3. The equilibrium point of the non-homogeneous difference equation

   $x_{t+1} = \alpha x_t + b, \alpha \neq 1$ is stable if <CO-2>

   i)$|\alpha| > 1$ ii) $|\alpha| < 1$ iii) $\alpha \neq 0$ iv) none

4. The number of stable equilibrium point/s of the system

   $x_{t+1} = \frac{rx_t^2}{x_t^2+A}$ $(r > 0, A > 0, r > 2\sqrt{A}\,)$ <CO-3>

   i) 0 ii) 1 iii) 2 iv) 3

5. The equilibrium point of the system

   $\frac{dx}{dt} = y,$

   $\frac{dy}{dt} =(\alpha - 1)x - \alpha y, \alpha \neq 1, \alpha > 0$ is stable if <CO-3>

   i)$\alpha = 2$ ii) $0 < \alpha < 1$ iii) $\alpha > 1$ iv) $1 < \alpha < 2$

6. The equilibrium point of the non-homogeneous difference equation $x_{t+1} = ax_t + b, a \neq 1$ is stable if i)$|a| > 1$ ii) $|a| < 1$ iii) $|a| < 2$ iv) none <CO-2>

7. Slope of the reproduction curve of the system

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$x_{t+1} = \frac{rx_t}{x_t+A}(r > 0, A > 0)$ at $x = 0$ is <Co-2>

i) 0 ii) A iii) $\frac{A}{r}$ iv) $\frac{r}{A}$

8. If $\bar{x}$ be an equilibrium point of the system $x_{t+1}=f(x_t)$,$x(0)=x_0$, then $\bar{x}$ is stable if <Co-1>

 i) $f'(\bar{x}) < 0$ ii) $|f'(\bar{x})| = 1$ iii) $|f'(\bar{x})| > 1$ iv) $|f'(\bar{x})| < 1$

9. The number of stable equilibrium point/points of the system $x_{t+1} = \frac{rx_t^2}{x_t^2+A}(r > 0, A > 0, r > 2\sqrt{A})$ is <Co-3>

 i) 0 ii) 1 iii) 2 iv) 3

10. The nontrivial equilibrium point of the system
 $x_{t+1} = \frac{ax_t}{x_t+b}, a > 0, b > 0, a \neq b$ is locally asymptotically stable if <Co-3>

 i) $a < b$ ii) $a > b$ iii) $a = b$ iv) $a = 1$

11. The non-trivial equilibrium point of the discrete logistic model

$$x_{t+1} = (1+r)x_t - \frac{rx_t^{\,2}}{K}, r > 0, K > 0$$

<Co-3>

 is locally asymptotically stable if

 i) $r = 2$ ii) $r > 1$ iii) $0 < r < 2$ iv) $r > 2$

12. The trivial equilibrium point (0,0) of the discrete growth model given by

$$x_{t+1} = x_t(a - x_t - y_t), a > 0$$
$$y_{t+1} = y_t(b + x_t), 0 < b < 1$$

 is locally asymptotically stable if <Co-3>

 i) $a = 0$ ii) $a < 1$ iii) $a = 1$ iv) none

13. The equilibrium point $(a-1,0)$ of the discrete prey-predator model given by

$$x_{t+1} = x_t(a - x_t - y_t), a > 0$$

 $y_{t+1} = y_t(b + x_t), 0 < b < 1$s <Co-3>

 is locally asymptotically stable if

 ii) $1 < a < b$ ii) $1 < a < 2 - b$ iii) $0 < a < 1$ iv) none

14. The trivial equilibrium point (0,0) of the Nicholson-Bailey model given by

$$H_{t+1} = bH_t e^{-aP_t}$$
$$P_{t+1} = cH_t(1 - e^{-aP_t})$$

<Co-3>

 is locally asymptotically stable if

 i) $b < 1$ ii) $a < 1$ iii) $b = 1$ iv) $a = 1$

15. The non- trivial equilibrium point $(\bar{H}, \bar{P})$ of the Nicholson-Bailey model given by

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$$H_{t+1} = bH_t e^{-aP_t}$$
$$P_{t+1} = cH_t(1 - e^{-aP_t})$$

i) is always stable ii) is always unstable iii) is stable if $b > 1$ iv) is stable if $b = 1$ <CO-2>

16. The steady state $x^* = 3$ of the difference equation $x_{n+1} = x_n e^{3-x_n}$ is
    i) asymptotically stable ii) stable but not asymptotically stable iii) unstable iv) none <CO-2>
17. The fixed point $x^* = \frac{(\alpha-1)}{\beta}$ of the difference equation $x_{n+1} = \frac{\alpha x_n}{1+\beta x_n}, \alpha > 1, \beta > 0$
    ii) Unstable ii) asymptotically stable iii) stable but not asymptotically stable iii) unstable iv) none <CO-3>
18. The equilibrium point (3,0) of the following two dimensional model

$$\frac{dx}{dt} = x\left(1 - \frac{x}{3}\right) - xy$$

<CO-3>

$$\frac{dx}{dt} = y(x-1)$$

is a i) stable node ii) unstable saddle point iii) locally asymptotically stable iv) none

19. The Holling type-III functional response $\varphi(N)$ represents in $(N, \varphi(N))$ plane
    i) a sigmoidal curve ii) a closed curve iii) a hyperbolic curve iv) a straight line. <CO-1>
20. In Gompertz growth model $\frac{dP}{dt} = CPlog(K/P)$, the population$(P)$ grows fastest when P is equal to i) 0 ii) K iii) $\frac{e}{K}$ iv) $\frac{K}{e}$, C, K being positive parameters. <CO-2>
21. What type of bifurcation will occur in the system $\frac{dx}{dt} = \mu x - x^3$, where μ is bifurcation parameter?
    i) Saddle-node bifurcation ii) pitchfork bifurcation iii) transcritical bifurcation iv) none <CO-2>
22. A two-dimensional system has the characteristic equation $\lambda^2 + \alpha\lambda + \alpha\beta(1-\alpha) = 0$ (where $\alpha > 0, \beta > 0$) at the equilibrium point $(\alpha, 1-\alpha)$ if then the equilibrium point is a i) unstable focus ii) unstable node iii) stable focus iv) stable node. <CO-3>
23. The equilibrium point $x^* = k$ for the equation $\frac{dx}{dt} = rx(1 - \left(\frac{x}{k}\right)^\theta)$ where r, k, θ are positive parameters is
    i) Unstable ii) stable iii) stable but not asymptotically stable iv) none. <CO-3>
24. Consider a dynamical system $\frac{dr}{dt} = r(1-r)(r-2)(r-3), \frac{d\theta}{dt} = 1$ where (r, θ) be the polar coordinates on the plane,
    The number of limit cycles is i) 1 ii) 2 iii) 3 iv) 4. <CO-3>
25. The fixed point $x^* = \frac{\alpha-1}{\beta}$ of the difference equation $x_{n+1} = \frac{\alpha x_n}{1+\beta x_n}, \alpha > 1, \beta > 0$ is
    i)unstable ii) asymptotically stable iii) stable but not asymptotically stable iv) none. <CO-3>

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26. The equilibrium point (0,0) of the system $\dot{x} = y, \dot{y} = (\alpha - 1)x - y$ where $0 < \alpha < 1$ is i) unstable node ii) stable node iii) saddle point iv) center <CO-3>

27. The equilibrium point of the system $\dot{x} = y, \dot{y} = (\alpha - 1)x - \alpha y$ where $\alpha \neq 1, \alpha > 0$ is stable if i) $\alpha = 2$ ii) $0 < \alpha < 1$ iii) $\alpha > 1$ iv) $\alpha > 2$ <CO-3>

28. The system <CO-3>

$$\dot{x} = -y + x(1 - x^2 - y^2),$$
$$\dot{y} = x + y(1 - x^2 - y^2),$$

has i) stable limit cycle ii) unstable limit cycle iii) no limit cycle iv) limit cycle which may or may not be stable. <CO-3>

29. The equilibrium point (0,0) of the system

$$\dot{x} = -2x + 3y + xy$$
$$\dot{y} = -x + y - 2xy^2$$

<CO-3>

is i) stable node ii) saddle point iii) stable focus iv) unstable node

30. In the following Chemostat model

$$\frac{dx}{dt} = (K(c) - D)x$$

<CO-3>

$$\frac{dc}{dt} = D(c_0 - c) - \frac{1}{p} K(c)x,$$

The equilibrium point $(0, c_0)$ is stable if

i) $D < K(c_0)$ ii) $D > K(c_0)$ iii) $D/2 < K(c_0)$ iv) $2D < K(c_0)$

<CO-3>

Long Questions (Each Question carries 2/3/5 marks)

1. What do you mean by predator-prey interaction? <CO-1>
2. State the basic assumptions of classical Lotka-Volterra model for a predator-prey system. <CO-1>
3. Find the equilibrium points and the solve thefollowing classical Lotka-Volterra predator-prey model <CO-3>

$$\frac{dx}{dt} = x(\lambda - by), \qquad \frac{dy}{dt} = y(-\mu + cx),$$

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where λ, μ, b and c are positive constants, $x(t)$ and $y(t)$ are respectively the prey and predator population size at time t.

4. Find the equilibrium points and discuss their stability behaviour of the following classical Lotka-Volterra predator-prey system

$$\frac{dx}{dt} = x(\alpha - \beta y), \frac{dy}{dt} = y(-\gamma + \delta x),$$ <Co-3>

where α, β, γ and δ are positive parameters, $x(t)$ and $y(t)$ are respectively the prey and predator population size at time t.

5. Investigate the stability of the equilibrium points of the following modified Lotka-Volterra system

$$\frac{dx}{dt} = x(a - bx - cy), \frac{dy}{dt} = y(-k + dx),$$ <Co-4>

where $x(t)$ and $y(t)$are the prey and predator population sizes at time t; a, b, c, d and $k$are positive constants.

6. Find the equilibrium point of the following modified Lotka-Volterra system and discuss the stabilities of the equilibrium points

$$\frac{dx}{dt} = ax(k - x) - bxy, \qquad \frac{dy}{dt} = -cy + dxy,$$ <Co-3>

where $x(t)$ and $y(t)$ are the prey and predator population sizes at time t;a, b, c, d are positive constants, the carrying capacity of the prey population $k > {}^{c}/_{d}$.

7. Consider the prey-predator system

$$\frac{dN}{dT} = rN\left(1 - {}^{N}/_{k}\right) - cNP, \ \frac{dP}{dT} = -mP + bNP,$$ <Co-3>

where N is the prey population size, P , that of predators at any time T; $r(> 0)$be the growth rate and $k(> 0)$ is the carrying capacity of prey population and b, c, m are positive parameters.

a) Using the dimensionless quantities: $x = {}^{N}/_{k},\ y = {}^{cP}/_{r},\ t = rT$, deduce the following
dimensionless form of the above system

$$\frac{dx}{dt} = x(1 - x - y), \qquad \frac{dy}{dt} = \beta(x - \alpha)y$$

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where $\alpha = \frac{m}{bk}, \beta = \frac{bk}{r}$.

b) For the dimensionless system, find the equilibrium points and discuss their stabilities.

8. Consider the following classical Lotka-Volterra Competitive model
$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1}{k_1} - \frac{\alpha_{12}}{k_1} x_2\right), \quad \frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{k_2} - \frac{\alpha_{21}}{k_2} x_1\right),$
where $r_1, k_1, r_2, k_2, \alpha_{12}$ and α_{21} are positive constants; r_1, r_2 and k_1, k_2 be the growth rates and carrying capacities of the two species respectively; α_{12} and α_{21} are the competition coefficients of the species 2 on the species 1 and of species 1 on species 2 respectively; $x_1(t)$ and $x_2(t)$ are the population sizes of the two species respectively.
a) Find the zero-growth isoclines.
b) show that the 2^{nd} species will be going to extinction and the 1^{st} species approaches its carrying capacity if $\alpha_{12} < {}^{k_1}/_{k_2}$ and $\alpha_{21} > {}^{k_2}/_{k_1}$, but the opposite phenomenon occurs when $\alpha_{12} > {}^{k_1}/_{k_2}$ and $\alpha_{21} < {}^{k_2}/_{k_1}$.
c) Show that there exists a saddle point if $\alpha_{12} > {}^{k_1}/_{k_2}$ and $\alpha_{21} > {}^{k_2}/_{k_1}$ and a stable node in the interior of the 1^{st} quadrant if $\alpha_{12} < {}^{k_1}/_{k_2}$ and $\alpha_{21} < {}^{k_2}/_{k_1}$. <Co-4>

9. In the following Lotka-Volterra competitive model:

$$\frac{dx}{dt} = x(1 - x - \alpha y), \qquad \frac{dy}{dt} = \rho y(1 - y - \beta x),$$

where α, β and ρ are positive constants, $x(t)$ and $y(t)$ are the population sizes at time t.

a) Find the steady states.
b) Show that if $\alpha < 1$ and $\beta < 1$, then coexistence equilibrium point is asymptotically stable
but if $\alpha > 1$ and $\beta > 1$ then coexistence equilibrium point exists but unstable.
c) Show that if $\alpha > 1$ and $\beta < 1$, the 2^{nd} species will be surviving to its carrying capacity and the 1^{st} species will be going to extinction. <Co-3>

10. Determine the equilibrium points and discuss their stability behaviour of the following predator-prey system

$$\frac{dx}{dt} = x\left(1 - \frac{x}{13}\right) - \frac{xy}{x+10}, \quad \frac{dy}{dt} = y\left(\frac{x}{x+10} - \frac{3}{5}\right).$$ <Co-3>

11. Determine the equilibrium points of the following competitive model

$$\frac{dx}{dt} = x(100 - 4x - y), \quad \frac{dy}{dt} = y(60 - x - y).$$ <Co-3>

Also discuss the stability behaviour of the equilibrium points.

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12. Show that the equilibrium point (x^*, y^*) with $x^* > 0, y^* > 0$ of the predator-prey model

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{axy}{x + A}, \quad \frac{dy}{dt} = sy\left(\frac{ax}{x + A} - \frac{aB}{A + B}\right),$$

where r, s, a, A and B are positive constants, is unstable if $k > A + 2B$ and asymptotically stable if $B < k < A + 2B$. <CO-3>

13. Investigate the stability of the non-trivial equilibrium point of the following Lotka-Volterra competitive model

$\frac{dx_1}{dt} = x_1(16 - 2x_1 - x_2)$, $\frac{dx_2}{dt} = x_2(12 - x_1 - x_2)$. <CO-3>

14. Let (x^*, y^*) be an equilibrium point of the following system

$$\frac{dx}{dt} = f(x, y), \qquad \frac{dy}{dt} = g(x, y),$$

where f and g are continuously differentiable functions. <CO-3>

a) Obtain the corresponding linearized system about (x^*, y^*).

b) Hence discuss the stability of (x^*, y^*).

15. a) Write the Routh-Hurwitz criterion of stability for a n-dimension system.

b) Also deduce this for three-dimension and two-dimension system. <CO-2>

16. What is a compartmental model? State the basic assumptions of Kermack-Mckendrick SIR compartmental model. Draw the flow chart and write the model equations. Find the basic reproduction number. Determine the conditions for which epidemic spreads and infection dies out. <CO-1,2>

17. Suppose $x^*$is a fixed point of the system $x_{n+1} = f(x_n)$, where $f(x)$ is a continuous differentiable function and$f'(x^*) \neq 1$. prove that is asymptotically stable if $|f'(x^*)| < 1$ and unstable if $|f'(x^*)| > 1$. <CO-2>

18. Consider the following non-linear difference equation $x_{n+1} = \frac{\lambda x_n}{\mu + x_n}$ where $\lambda > 0. \mu > 0$. <CO-2>

Find the fixed points and discuss their stability.

19. Solve the following non-homogeneous system and discuss the stability of the fixed point by using Cobweb-diagram:

$$x_{n+1} = \frac{3}{4}x_n + 10.$$

<CO-3>

20. Consider the discrete-time predator-prey system

$$x_{n+1} = ax_n(1 - x_n) - bx_n y_n$$

$$y_{n+1} = -cy_n + dx_n y_n$$

<CO-3>

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Where a, b, c, d are positive parameters. Find the fixed points of the system and discuss their stabilities.

21. State the basic assumptions of spruce budworm population dynamics and construct the model equation with logistic growth and suitable predation term. Derive the corresponding dimensionless equation. <CO-3>

22. Consider the following epidemic model:

$$\frac{dS}{dt} = A - rS - \frac{\beta SI}{1+\alpha I}$$

$$\frac{dI}{dt} = \frac{\beta SI}{1+\alpha I} - \mu I,$$ <CO-3>

Where are positive parameters. Find the equilibrium points of the system.

23. Write short notes on the following
    i) Gompertz growth ii) Basic reproduction number. <CO-1>

24. Consider the growth model
$\frac{dN}{dt} = rN\left(\frac{N}{A} - 1\right)\left(1 - \frac{N}{K}\right)$, where r, A, K are positive parameters and $A < K$. Determine the equilibrium points and discuss their stability. <CO-3>

25. Find analytical solution of the following system

$$\dot{x} = -y + x(1 - x^2 - y^2)$$
$$\dot{y} = x + y(1 - x^2 - y^2)$$ <CO-3>

and then obtain the limit cycle of the system.

26. Show that the following system has a stable limit cycle

$$\dot{x} = y + \frac{x}{\sqrt{x^2+y^2}}(1 - x^2 - y^2)$$
$$\dot{y} = -x + \frac{y}{\sqrt{x^2+y^2}}(1 - x^2 - y^2)$$ <CO-3>

27. Find the limit cycle of the system

$$\dot{x} = (x^2 + y^2 - 1)x - y\sqrt{x^2+y^2}$$

$$\dot{y} = (x^2 + y^2 - 1)y + x\sqrt{x^2+y^2}$$ <CO-3>

and investigate its stability.

28. Define functional response in prey-predator interaction. Describe different types functional response in mathematical form and draw the sketches of the response curves. Explain the merits and demerits if any of different Holling type response functions. <CO-1,2>

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29. What are the defects of the Malthusian growth model? For a certain microorganism, birth is by budding of a fully copy of itself. Suppose that under reasonable favorable conditions , such birth occur on average four times per day and an individual lives on average ,one day. Write the differential equation for the population$p(t)$ of the organism as a function of time. Also find the solution, given that at time$t = 0$ the population size is 1000. <Co-1,3>
30. Write Short note on i) transcritical bifurcation ii) Hopf bifurcation. <Co-1>
31. Consider the Nicholson-Bailey model

$$N_{t+1} = rN_t e^{-aP_t}$$
$$P_{t+1} = cN_t(1 - e^{-aP_t})$$

Where the symbols have their usual meanings. Find the equilibria of the system and discuss their stabilities. <Co-2>

32. The population dynamics of a species is governed by the discrete model

$$N_{t+1} = N_t \exp\left(r\left(1 - \frac{N_t}{K}\right)\right)$$

where r, K are positive parameters. <Co-3>

i) Determine the steady states and their stability nature.
ii) Show that a period doubling bifurcation occurs at$r = 2$.

33. A drug is administered every six hours. Let D_n be the amount of drug in the blood system at interval. The body eliminates a certain fraction p of the drug during each time interval. If the initial drug D_0 ,find D_n and also find

$$\lim_{n\to\infty} D_n$$

<Co-3>

34. From the following SIR model

$$\frac{dS}{dt} = bK - \beta SI - \mu S$$
$$\frac{dI}{dt} = \beta SI - \mu I - \gamma I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

<Co-3>

Show that if $\beta K < (\mu + \gamma)$ there is only one equilibrium which is locally asymptotically stable. Also prove that if $\beta K > (\mu + \gamma)$ there is another equilibrium which is also asymptotically stable.

Question Bank on C-Programming Language

Subject-MTMA **SEM-I** **Paper- SEC-A**

MCQ (Each Question carries 2 marks)

(Each question below is followed by four possible answers of which exactly one is correct. Choose the correct answer with proper justification.)

1. What will be the output of the following c code? <CO-2>
```
{ int x=15,y=15;
x=x++;
y=++;
print f ( "%d%d\n",x,y);
}
```
   a) 10,15 b) 10,16 c) 11,16 d) 11,15
2. What will be the output of the following c code? <CO-2>
```
int k;
for(k=1, k>=10, k++);
```
   a) 11 b) 1 c) 10 d) 0
3. What will be the output of the following c code? <CO-2>
```
int main()
{
int a[5]={ 5,1,5,20,25};
int i, j, m;
i=++ a[1];
j=a[1]++;
k=a[i++];
print f(" %d %d%d", i,j,m);
return 0;
}
```
   a) 2,1,15 b) 1,2,5 c) 3,2,15 d) 2,3,20.
4. Study the following statement <CO-2>
```
for ( i=3;i<10;i++)
print f("%d",++i);
```
   What will be the output ?
   a) 3,5,7,9 b) 4,6,8,10 c) 3,7,11 d) none
5. Which one of the following is invalid real constant?

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<Co-2>

a) 0.0083 b) 2E+10.2 c) 435.76 iv) +247.0

6. What will be the output of the following c code?

```
# include <stdio.h>
main ()
{
int  a =10/3;
print f( "%d",a);
```

<Co-2>

a) 3 b) 3.333333 c) no output will be shown d) 10/3;

7. What will be the output of the following c code?

```
void main ()
{
    int x=3 ,y=0;
    if (x!=y)
    print f( " Not Equal");
    else
    print f( "  Equal");
```

<co-1>

a) Equal b) Not Equal c) Not Equal Equal d) Equal Not Equal

8. What will be the output of the following c code?

```
#include<stdio.h>
main ()
{
int i=-3,j=2,k=0,m;
m=++i&&++j&&++k;
print f(%d %d %d %d\n", i, j, k, m);
}
```

<Co-2>

a) -2, 3, 1, 1 b) 2, 3, 1, 2 c) 1, 2, 3, 1 d) 3, 3, 1, 2

9. The output of the following program is

```
int x=0;
int main()
{
int x=0;
printff( "%d",x);
return 0;
}
```

<Co-1>

a) 0 b) 10 c) no output d) none

10. What will be the output of the following c code?

```
# include <stdio.h>
Void main ()
{
int x, y=5, z=5;
```

<Co-1>

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```
x=y= =z;
print f( "%d",x);
}
```

a) 0 b) 1 c) 5 d) Error

11. How many times will be the following loop execute? <co-2>

```
for (j=1; j<=10; j=j-1)
```

a) Forever b) Never c) 0 d) 1

12. What will be the output of the following c code? <co-2>

```
int a=4, b=6;
printf("%d", a!=b);
```

a) Output an error message b) prints 0 c) prints 1 d) none

13. What will be the output of the following c code? <co-2>

```
int i=5;
int j=i/-4;
int k=i%-4;
printf( "%d%d",j ,k);
```

a) 1,1 b) -1,-1 c) -1,1 d) 1, -1.

14. What will be the output of the following c code? <co-2>

```
#include<stdio.h>
int main ()
{
int c=1;
int s=0;
while ((c>0)&&(c<60))
{
S=s+c;
C++;
}
print f("d" ,s);
```

a) 1771 b) 1770 c) 1772 d) none

15. What will be the output of the following c code? <co-2>

```
int main ()
{
int a[5] = { 5,1,15,20,25};
int i, j, m;
i = ++ a[1];
j = a[1]++;
```

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```
k =a[++i];
print f("%d%d%d", i,j,k);
```

<Co-1>

a) 2,1,15 b) 1,2,5 c) 3,2,15 d) 2,3,20

16. What will be the output of the following C code?

```
#include <stdio.h>
void main()
{
int x, y = 5, z = 5;
x = y = = z;
printf("%d", x);
}
```

<Co-1>

a) 0 b) 1 c) 5 d) error

17. Which one of the following operators is used for decision making in C?

<Co-1>

a) Arithmetic operator b) Relational operator

c) Assignment operator d) Conditional operator

18. In which order the following gets evaluated?

A) Arithmetic B) Assignment C) Logical D) Relational

<Co-1>

a) (B)→ (A) →(D) → (C) b) (B)→ (D) →(A) → (C)

c) (B)→ (C) →(D) → (A) d) (B)→ (D) →(C) → (A)

19. Which one of the following is invalid real constant?

<Co-2>

a) 0.00833 b) 2E+0.2 c) 435.76 d) +247.0

20. What is the output of the following C program?

```
# include < stdio.h >
main()
{
int a=10/3;
printf("%d", a);
}
```

<Co-2>

a)3 b) 3.333333 c) no output d) 10/3

Answer:

1. b) 2. b) 3. a) 4. b) 5. b) 6. a) 7. b) 8. a) 9. a) 10. b) 11. a) 12. c) 13. c) 14. b) 15. c) 16.b) 17. b) 18.c) 19.b) 20.a)

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Long Questions: (SEC-A) (Hons) (Question carrying 2/3/4/ 5/6 marks)

1. Write an algorithm to find all roots (real/complex) of a quadratic equation $ax^2 + bx + c = 0$. Also construct the corresponding flowchart. 3+2 ⟨Co-3⟩

2. Write a c program to check whether a number prime or not 5 ⟨Co-3⟩

3. Write a program to swap two numbers without using third variables. 3 ⟨Co-3⟩

4. Using recursion write a program to evaluate factorial of n. ⟨Co-3⟩
5. Draw a flowchart to print the factors of the integer 215 and find the sum of the factors. 5 ⟨Co-3⟩

6. Write an algorithm to find H.C.F. and L.C.M of two distinct positive integers m and n. 5 ⟨Co-3⟩

7. Draw a flowchart to print the first 20 terms of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, 21,.............................. ⟨Co-3⟩ 5

8. Write a c program to calculate the value of sin (x) from the power series expansion (considering first five terms). ⟨Co-4⟩ 5

9. What is operating System? Give Example. State two functions of operating system 2+1+2 ⟨Co-1,3

10. Write down the while and do-while statement which are equivalent to the following for statement: ⟨Co-3⟩

```
for(n=1; n<=10; ++n)
{
............
............
}
```

5 ⟨Co-2⟩

11. Write short notes on Compiler ,Interpreter and Assembler . 2+2+1 ⟨Co-2⟩

12. What is the difference between Call by value and Call by address ? 5 ⟨Co-3⟩

13. What is the output of the following C code ?

```
# include <stdio.h>
main ( )
{
```

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```
int x , y ;
y=6 ;
x = y <<6 ;
print f ( " % d", x) ;
}
```

3

14. Write a C program to find sum of first n odd positive integers to illustrate call by value. <CO-3>

5

15. What is meant by an operator in C? Explain the functions of logical AND and logical OR operators. <CO-3>

1+2+2

16. (i) How can you use *break* and *continue* statements in for loop? Give suitable example to justify youranswer. <CO-4>

    (ii) Write a C-program to test whether a number is prime or not. (2+3)+5

17. (i) Write the benefits of using functions in C. Distinguish between the user-defined function andstandard build-in functions. <CO-4>

    (ii) Write a C-program to find the functional values for five given values of x, where

    $f(x) = x^2 + \sin(x)$, $0 \le x < 2$.

    $= 2\cos(x) - 1$, $2 \le x \le 4$

    and input values of x are $0{\cdot}2,\ 1{\cdot}8,\ 2{\cdot}0, 2{\cdot}5, 3{\cdot}5$ (2+3)+5

18. (i) Explain conditional operator using suitable example. What are the limitations of conditionaloperator? <CO-4>

    (ii) Write an algorithm to find factorial of a given number. Hence write the corresponding C-program.

    [(2+2)+(3+3)] <CO-3>

19. (i) Write down the syntax of for loop in C and draw the corresponding flow diagram.

    (ii) Write a C program to print $a = 10, 11, 12, 13, 14, 15, 16, 17, 18, 19$ using for loop. 2+2+6

    <CO-3>

20. (i) Write a C-program to find the arithmetic mean of n real numbers.

    (ii) Write a C-program to find the sum of the series:

    $1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \ldots\ldots\ldots\ldots$ correct to three decimal places at $x = 0.5$. <CO-3>

    5+5

21. i) What is Mixed-mode Arithmetic? Explain with an example.

    (ii) Using Integer Arithmetic write a C-program to convert the given number of days into months and daysand print the result. <CO-3>

    2+2+6

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22. (i) What do you mean by one dimensional array? Give an example. <CO-3>

(ii) Using array write a C-program to sort a given set of numbers in descending order. 2+2+6

23. (i) Discuss the difference between library functions and user defined functions with suitable examples. <CO-3>

(ii) Write a C-program to compute and print a multiplication table for numbers 1 to 5 as shown below:

	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

<CO-3>

using two-dimensional array. 5+5

24. (i) Write about the following errors in C: Syntax error; Run-time error; Logical error. <CO-2> <CO-3>

(ii) Write an algorithm and draw the flow chart for finding the real roots of $ax^2 + bx + c = 0$.

[(2+2+2)+4] <CO-3>

25. (i) Write down the syntax of if-else statement and draw the corresponding flow chart. <CO-2>

(ii) Using if-else statement, write a C program to check whether the entered age is greater than or equal to 18 (years). If this condition meets then display the message, "You are eligible for voting"; however if the condition does not meet then display the message, "You are not eligible for voting". <CO-3>

(2+2+6) <CO-3>

26. (i) What is local variable and global variable? Explain with suitable example. <CO-2>

(ii) Distinguish between RAM and ROM. <CO-2>

(iii) Write a C-program to find the sum of the digits of a number. [3+2+5]

27. (i) What is meant by 'Nesting of Functions' in C? <CO-3>

(ii) Is the following C-program an example of Nesting of Functions? Explain your answer logically:

```
#include <stdio.h>
int difference(int p, int q)
{
```

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```
        if(p!=q)
        return( 1);
        else
        return (0);
}
float ratio(int x, int y, int z)
{
        if(difference(y,z))
        return((x/(y-z)));
        else
        return (0.0);
}
        int main()
{int a, b, c;
        float ratio (int a, int b, int c);
        scanf("%d%d%d", & a, & b, & c);
        printf ("%f\n", ratio(a, b, c));return
        0;
}
```

(iii) What is recursion in C? Explain with an example.

2+5+3 <CO-4>

28. i)What is meant by a variable in C language? Explain with a suitable example. <CO-2>

ii)State the conditions that a variable name in C must satisfy.

iii)What is the general form of exponential notation for a real constant? Explain the <CO-2>
terms

usedin the general form with a suitable example.

2+5+(1+2)

29. (i) What is meant by an operator in C? Explain the functions of logical AND and logicalOR operators. <CO-2>

(ii) Draw the flowchart for the following program segment : <CO-2>

.....

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```
.....
if  (category=A)
{
   marks = marks+bonus_marks;
}
printf("%f", marks);
......
......
```

(2+2 +2)+4

30. a) What is recursion? Write a function to evaluate factorial of *n* to show how the recursion works.

2+6+2 <CO-2>

31. (i) What are the necessary header files to be included in the following program to get the properoutput? Justify your answer.

```
void  main()
{
int  number,
i;sum=0;
printf("\n  enter  the  number");
scanf("%d", & number);
for  (i=1;  i<=number;  i++)
      {
         sum=sum+(1/pow(i,2));
         if(i==1)
            printf("\n1+");
             else
              if  (i==number)
            printf("1/(%d)^2", i);
            else
            printf("1/(%d)^2+", i);
      }
}
```

<CO-3>

(ii) State the advantages of Library functions. <CO-2>

(iii) Can you recognize the series which is mentioned in the above program? <CO-3>

(1+1+2+2)+3+1

32. (i) What are the main data types in C? Write short notes on any two of them. <CO-2>

(ii) Write a C program to display the following output : (using nested for loop)

```
*
* *
```

<CO-4>

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* * *
* * * *
* * * * *

(2+2+2)+4 <CO-2>

33. Write a short note on Branching statements in C.
    (ii) Write a C program to add two matrices and print the resultant matrix.

4+6 <CO-3>

34. (i) What is the objective of the main()function in C? What is the purpose of printf() and
    scanf() in C program? <CO-1>
    (ii) How is a function declared in C Language? <CO-2>
    (iii) Write a program to swap two numbers without using the third variable. <CO-3>
    (2+2)+3+3

35. i)What are the advantages of using C language over other programming languages? What aresome of the limitations of C language? <CO-2>
    (ii) Write a C program to check whether a given number is prime or not. <CO-3>

(4+2)+4

36. (i) State the differences between the declaration of a variable and the definition of a symbolicname. <CO-2>
    (ii) Write the valid C-expressions :
    A) $e^{x^3+2cosx} + log|x^5+1|$ B) $\tan(x^3+1) + \frac{1}{cos(x)+\sec(x)}$ <CO-2>
    (iii) X=2020, $y = 2021$, $z = 2022$. Write a C-program to rotate their values such that x hasthe value y, y has the value z and z has the value x.

3+3+4

37. (i) Write the syntax of for-loopand explain it. <CO-2>
    (ii) Using for-loop, write a program in C to find first 50 Fibonacci numbers. <CO-3>
    (iii) What is do-whilestatement? Explain with suitable example. <CO-2>

3+4+3

38. (i) Describe the two ways of passing parameters to functions. When do you prefer to use eachof them? <CO-2>
    ii)Write a program that use a function to sort an array of integers. <CO-3>
    iii)What is the output of the following C-code? <CO-3>

```
#include<stdio.h>main()
{
int x,y;
y=6;
```

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```
x=y<<6
;
printf("%d",x);
}
```

(2+2)+4+2

39. (i) Distinguish between function Call by valuewith Call by Address. <CO-3>

(ii) Write a C-program to find sum of first n odd positive integers and to illustrate Call by value. <CO-3>

(2+2)+6

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Question Bank on Partial Differential Equation

Subject-MTMA **SEM-VI** **Paper-CC -9**

MCQ (Each Question carries 2 marks)

(Each question below is followed by four possible answers of which exactly one is correct. Choose the correct answer with proper justification.)

1. The Partial Differential Equation $\frac{\partial^2 z}{\partial x^2}$ -5 $\frac{\partial^2 z}{\partial y^2}$ =0 is classified as
   a) Elliptic , b) Parabolic , c) Hyperbolic , d) none <CO-1>
2. A Partial Differential Equation has
   a) One independent variable , b) Two or more independent variables , c) More than one dependent variable , d) Equal number of dependent and independent variables. <CO-1>
3. When Solving a one –dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by using separation of variable method we get the solution if <CO-2>
   a) k>0 , b) k<0, c) k=0, d) none
4. The solution of $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y}$ =u with u(o,y) =$e^{2/y}$ is <CO-3>
   a) $e^{-3/y}.e^{2/x}$, b) $e^{3/y}.e^{2/x}$, c) $e^{-3/x}.e^{2/y}$, d) $e^{3/x}.e^{2/y}$
5. The Solution of xp +yq =z is
   a)$f\left(\frac{x}{y},\frac{z}{y}\right) = 0$, b) $f\left(\frac{y}{x},\frac{y}{z}\right) = 0$, c) $f\left(\frac{x}{y},\frac{y}{z}\right) = 0$, d) $f(x,y) = 0$ <CO-3>
6. The Partial Differential Equation of the family $z = (x-a)^2 + (y-b)^2$, where a and b are arbitrary constants is
   a) $p^2 + q^2 = 4z$, b) $p^2 - q^2 = 4z$, c) $p^2q^2 = 4z$, d) none. <CO-3>
7. The Complete integral of z(xp-yq) = $y^2 - x^2$ is
   a) $x^2 + y^2 + f(xy) = z^2$, b) $x^2 - y^2 + f(xy) = z^2$, c) $-x^2 - y^2 + f(xy) = z^2$, d) $-x^2 + y^2 + f(xy) = z^2$. <CO-3>
8. The general solution of $(x^2 + y^2 + z^2)$p -2xyq + 2xz = 0 is
   a) $\emptyset\left(\frac{y}{z},\frac{x^2+y^2+z^2}{z}\right) = 0$,b) $\emptyset\left(\frac{x}{z},\frac{x^2+y^2+z^2}{y}\right) = 0$, c) $\emptyset\left(\frac{y}{z},\frac{x^2-y^2+z^2}{z}\right) = 0$, d) $\emptyset\left(\frac{x}{z},\frac{x^2-y^2+z^2}{z}\right) = 0$ <CO-3

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9. The Solution of $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}$ =0 is of the form

   a) z= $(c_1cospx + c_2\,sinpx)(c_3e^{py} + c_4\,e^{-py})$,b) z= $(c_1e^{py} + c_2\,e^{-py})$,c) z = $(c_1cospx + c_2\,sinpx)$,d) z= $cospx(c_3e^{py} + c_4\,e^{-py})$, p is constant. <CO-3>

10. The D'Alembert's Solution of the wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\,\frac{\partial^2 u}{\partial x^2}$

    a) $f_1(x-ct)$+ $f_2(x+ct)$, b) $f_1(x)$+ $f_2(x+ct)$,c) $f_1(ct)$+ $f_2(x+ct)$,d) $f_1(x-ct)$+ $f_2(ct)$ <CO-3>

11. The Partial Differential Equation obtained from $x^2 + y^2 + (z-c)^2 = a^2$ is

    a) x/p – y/q = 0 ,b) yp – xq =0 , c) xq – y/p =0 ,d) xy – pq =0. <CO-3>

12. Let $\vec{F}$ = ay$\hat{\imath}$ + z$\hat{\jmath}$+ x$\hat{k}$ and c be the positively oriented closed curve given by $x^2 + y^2 = 1, z = 0$. If $\oint_c \vec{F}.d\vec{r}= \pi$ then the value of a is

    a) -1, b) 0 , c) ½ , d) 1 <CO-2>

13. Consider the vector field $\vec{F}$ = (ax +y +a)$\hat{\imath}$ +$\hat{\jmath}$-(x +y) $\hat{k}$, where a is constant. If $\vec{F}.curl\vec{F} =$ 0,

    then the value of a is

    a) -1 , b) 0 , c) 1 , d) 3/2 <CO-2>

14. For c >0 ,if a$\hat{\imath}$ + b$\hat{\jmath}$+ c$\hat{k}$ is the unit normal vector at (1,1,$\sqrt{2}$) to the cone z =$\sqrt{x^2 + y^2}$, then

    a) $a^2 + b^2 + c^2 = 0$, b) $a^2 - b^2 + c^2 = 0$,c) $-a^2 + b^2 + c^2 = 0$, d) $a^2 + b^2 - c^2 = 0$ <CO-3>

15. The value of c for which there exist a twice differentiable vector field $\vec{F}$ with curl$\vec{F}$ = 2x$\hat{\imath}$ -7y$\hat{\jmath}$+ cz$\hat{k}$ is

    a) 0, b) 2, c) 7, d) 5 <CO-3>

16. The solution of $x^3\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y}$ =0 with u(o,y) =$10e^{5/y}$ is

    a) $10\,e^{5/y}.e^{5/2x^2}$, b) . $10\,e^{5/x}.e^{-5/2y^2}$ c) $10\,e^{-5/x}.e^{-5/2y^2}$, d) $10\,e^{5/y}.e^{-5/2x^2}$ <CO-3>

17. The solution of $\frac{\partial u}{\partial x} = 36\frac{\partial u}{\partial t}$ +10u with u(x,0) =$3e^{-2x}$ is

    a) -3/2 $e^{-2x}.e^{-t/3}$, b) 3 $e^{x}.e^{-t/3}$, c) 3/2 $e^{2x}.e^{-t/3}$, d) 3 $e^{-x}.e^{-t/3}$, <CO-3>

18. The Partial Differential Equation of $z = f(x/y)$, where f is arbitrary is

    a) $px = qy$, b) $px = -qy$, c) $py = qx$, d) $py = -qx$. <CO-3>

19. The Partial Differential Equation A $\frac{\partial^2 z}{\partial x^2}$ +2H $\frac{\partial^2 z}{\partial x \partial y}$ $+B\frac{\partial^2 z}{\partial y^2} + 2F\frac{\partial z}{\partial x} + 2G\frac{\partial z}{\partial y}$ +Cz =0 is of Elliptic

    type if a) AB$-H^2 > 0$, b) AB$-H^2 < 0$, c) AB$-H^2 = 0$, d) none. <CO-2>

20. The Solution of $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ if u(0,t)= u(3,t)=0 and u(x,0) = $5sin4\pi x - 3sin8\pi x$ is <CO-3>

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a) $c_1\, sinpt(c_3 cospx + c_4 sinpx)$, b)$-\frac{1}{t^2} sinxt + xf_1(t) + f_2(t)$,

b) $(5sin4\pi x - 3sin8\pi x)e^{\frac{-2\pi^2 x^2 t}{9}}$,d) $(sin4\pi x + 3sin8\pi x)e^{\frac{-2\pi^2 x^2 t}{9}}$

21. The Solution of $\frac{\partial^2 z}{\partial x^2}$ =sin (xy) is

a) $-\frac{1}{y^2} sinxy + xf_1(y) + f_2(y)$, b) $sinxy + xf_1(y) + f_2(y)$, c) $-\frac{1}{y^2} sinxy + f_2(y)$, $d) sinxy + f_1(y) + y^2 f_2(y)$

<CO-3>

22. The Solution of $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$ if u(0,t)= u(l,t)=0 and u(x,0) = sin $(\frac{\pi x}{l})$, $\frac{\partial u(x,0)}{\partial t}$ =0 is

a) $\sin(\frac{\pi x}{l})\cos(\frac{\pi ct}{l})$,b) $-\sin(\frac{\pi x}{l})\cos(\frac{\pi ct}{l})$, c) $\cos(\frac{\pi x}{l})\sin(\frac{\pi ct}{l})$, d) none <CO-3>

23. The general solution of $(\frac{y^2 z}{x})$p +xzq = y^2 is

b) $\emptyset(x^3+y^3, x^2+z^2)=0$,b) $\emptyset(x^2-y^2, x^2-z^2)=0$, c) $\emptyset(x^3-y^3, x^2+z^2)=0$, d) $\emptyset(x^3+y^3, x^2-z^2)=0$, $\emptyset$ being an arbitrary function . <CO-3>

24. The Solution of zp=-x is

a) $x^2 - z^2 = \emptyset(y)$, b) $x^2 + z^2 = \emptyset(y)$,c) $x^3 + z^3 = \emptyset(y)$,d) $x^2 - y^2 = \emptyset(y)$ <CO-3>

25. The solution of x$\frac{\partial u}{\partial x}$ + y$\frac{\partial u}{\partial y}$ =4u satisfies u(x,y) = 1 on $x^2 + y^2$=1. Then u(2,2)= <CO-3>

a) 32 ,b) 64 , c) -64, d) -32

26. If u be the unique solution of $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$,$x \in \mathbb{R}$, t >0 , u(x,0) = x(1-x) for all x∈ [0,1] and f(x+1)=f(x) for all $x \in \mathbb{R}$ and $\frac{\partial u(x,0)}{\partial t}$=0 , $x \in \mathbb{R}$ then u(1/2, 5/4) is <CO-3>

a) 3/8, b) 3/2 , c) -3/2, d) 3/16

27. Let C be the boundary of the region in the first quadrant bounded by y=1$-x^2, x = 0$ and y=0 oriented counter-clockwise. The value of $\oint_c (xy^2 dx - x^2 y dy)$ is <CO-3>

a) 1/3, b) 2/3, c) -2/3 d) -1/3

28. Let S be the oriented surface $x^2 + y^2 + z^2 = 1$ with the unit normal vector $\hat{n}$ pointing outward. For the vector $\vec{F}$ = x$\hat{\imath}$ + y$\hat{\jmath}$+ z$\hat{k}$ the value of $\iint_S \vec{F}.\hat{n}$ dS is <CO-3>

a) $\frac{\pi}{3}$, b) 2 π , c) 4$\frac{\pi}{3}$, d) 4 π

29. Let R be the planer region bounded by the lines x=0 ,y=0 and the curve $x^2 + y^2 = 4$ in the first

quadrant. Let C be the boundary of R oriented counter-clockwise. Then the value of $\oint_c (x(1-y)dx + (x^2 - y^2)dy)$ is <CO-3>

a) ½, b) 0 , c) 7 , d) 8

30. Using Stoke's Theorem evaluate the line integral $\int_L$(y$\hat{\imath}$ + z$\hat{\jmath}$+ x$\hat{k}$). $d\vec{r}$ where L is the intersection

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of $x^2 + y^2 + z^2 = 1 \, and \, x + y = 0$ traversed in the clockwise direction when viewed from

the point (1,1,0) ,then the value is <CO-3>

a) $\sqrt{2}\,\pi$, b) π , c) $-\pi$, d) 0

31. The solution of $5\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y}$ =2u with u(o,y) =$9e^{-5y}$is

a) $9e^{17x/5}.e^{-5y}$, b) $9e^{13x/5}.e^{-5y}$. c) $9e^{-17x/5}.e^{-5y}$, d) $9e^{-13x/5}.e^{-5y}$ <CO-3>

32. The Solution of $\frac{\partial^2 z}{\partial x \partial y} = 0$ is

a) $z = f_1(x) + f_2(y)$,b) $z = xf_1(x) + yf_2(y)$, c) $z = f_1(x) - yf_2(y)$, d) $z = xf_1(x) - yf_2(y)$ <CO-2>

33. The Solution of PDE of the form $z = px + qy + f(p, q)$ can be written as

a) $z = ax + by + \sqrt{(a^2 + b^2)}$, b) $z = ax + by + \sqrt{(a^2 + 1)}$,

b) $z = ax + by + \sqrt{(b^2 + 1)}$, d) $z = ax + by + \sqrt{(1 + a^2 + b^2)}$. <CO-2>

34. The Solution of $a(p + q) = z$ is

a) $\emptyset(x - y, y + az) = 0$, b) $\emptyset(x + y, y - az) = 0$,c) $\emptyset(x - y, y - az) = 0$, d) $\emptyset(x + y, y + az) = 0$, $\emptyset$ being an arbitrary function. <CO-2>

35. The Solution of $xp - yq = xy$ is

a) $x = ce^{z/xy}$, b) $xe^{-z/xy} = \emptyset(xy)$, c) $xe^{z/xy} = \emptyset(xy)$, d) $xe^{-xy/z} = \emptyset(xy)$ <CO-2>

36. The solution of $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} -\infty < x < \infty, t > 0$ of D'Alembert's approach with $u(x, 0) = x^2, u_t(x, 0) = -x, -\infty < x < \infty$ is

a) $u = x^2 + t^2 + xt$, b) $u = x^2 - t^2 + xt$, c) $u = x^2 + t^2 - xt$, d) $u = x^2 + t^2 + x^2t^2$, <CO-3>

37. The Solution of $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ is

a) $\emptyset(x^2 + y^2 - z^2, xyz) = 0$,b) $\emptyset(x^2 - y^2 + z^2, xyz) = 0$, c) $\emptyset(x^2 + y^2 + z^2, xy) = 0$, d) $\emptyset(x^2 + y^2 + z^2, xyz) = 0$ <CO-3>

38. The Solution of PDE of the form $z = px + qy + p^2 + q^2$ can be written as

a)$z = ax + by + (a^2 + b^2)$, b) $z = ax - by + (a^2 - b^2)$, <CO-3>

c) $z = ax + by + (a^2 - b^2)$, d) $z = ax - by + (a^2 + b^2)$

39. The Canonical form of $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2} (c > 0)$ is <CO-3>

a) $\frac{\partial^2 z}{\partial u^2} = 0$, b) $\frac{\partial^2 z}{\partial u \partial v} = 0$, c) $\frac{\partial^2 z}{\partial v^2} = 0$, d) none

40. The Cauchy Problem $2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y}$ =5, $u(x, y) = 1$ on the line $3x - 2y = 0$ has

a) Exactly one Solution, b) Exactly two Solutions, c) Infinitely many Solutions , d) No Solution <CO-2>

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41. The Work done by the force $\vec{F} = 4y\,\hat{\imath} - 3xy\hat{\jmath} + z^2\hat{k}$ in moving a particle over the circular path $x^2+y^2 = 1, z = 0$ from (1.0,0)to (0,1,0) is <CO-3>
a) $\pi + 1$, b) $\pi - 1$, c) $-\pi + 1$, d) $-\pi - 1$,

42. Let $\vec{F} = 2z\,\hat{\imath} + 4x\hat{\jmath} + 5y\hat{k}$ and let C be the curve of intersection of the plane $z = x + 4$ and the cylinder $x^2+y^2 = 4$, oriented counter clockwise. The value of $\oint_c \overrightarrow{F}.d\vec{r}$ is <CO-3>
a) 4π, b) π, c) -4π, d) $-\pi$

43. Let W be the region bounded by the planes $x = 0, y = 0, y = 3, z = 0$ and $x + 2z = 6$. Let S be the boundary of this region. (Using Gauss' Divergence theorem) The value of $\iint_S \vec{F}.\hat{n}$ dS where $\vec{F} = 2xy\,\hat{\imath} + yz^2\hat{\jmath} + xz\hat{k}$ and $\hat{n}$ is the unit outward drawn normal vector to S is <CO-3>
a) 353/2, b) 351/2, c) 151/2, d) none

44. Let C be the boundary of the region $R = \{(x,y) \in R^2 : -1 \le y \le 1, 0 \le x \le 1 - y^2\}$ oriented in the counter clockwise direction. Then the value of $\int_C (ydx + 2xdy)$is <CO-2>
a) -4/3, b) -2/3 ,c) 2/3, d) 4/3

45. If C is a smooth curve in R^3 from $(-1,0,1) to (1,1,-1)$, then the value of $\int_C \{(2xy + z^2)dx + (x^2 + z)dy) + (y + 2xz)dz\}$ is <CO-2>
a) 0, b) 1, c) 2, d) 3

46. The PDE $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ is of type <CO-2>
a) Parabolic b) Hyperbolic c) Elliptic c) none

47. The solution of $\frac{\partial^3 z}{\partial x^2} = 0$ is <CO-3>
a) $z = f_1(x) + yf_2(x) + y^2 f_3(x)$ b) $z = f_1(y) + xf_2(y) + x^2 f_3(y)$
c)$z = (1 + x + x^2)f_1(y)$ d) none.

48. The solution of $u_{xx} = 9u_{yy}$ is
a) $u(x,y) = \sin(3x - y)$ b) $u(x,y) = 3x^2+y^2$ <CO-3>
c) $u(x,y) = x^2+3y^2$ d) $u(x,y) = \sin 3(x - y)$

49. The solution of $\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0, c > 1$
a) $u(x,y) = f(x + cy)$ b) $u(x,y) = f(x - cy)$ <CO-3>
c) $u(x,y) = f(cx + y)$ d) $u(x,y) = f(cx - y)$.

50. If $u(x,t)$ be the solution of $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$,$0 < x < \infty, t > 0$ satisfying

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$u(x,0) = \frac{cos\pi x}{2}, 0 \le x < \infty, u_t(x,0) = 0, 0 \le x < \infty, u_x(0,t) = 0, t > 0,$

then the value of $u(2,2)$ is

<(co-3)>

a) 0 b) -1 c) ½ d) 1

51. The integral surface for Cauchy Problem $\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} = 1$ which passes through the Circle $x^2 + y^2 = 1, u = 0$ is

<(co-2)>

a) $x^2 + y^2 + 2z^2 + 2zx - 2yz = 1$ b) $x^2 + y^2 + 2z^2 + 2zx + 2yz = 1$

c) $x^2 + y^2 + 2z^2 - 2zx - 2yz = 1$ d) $x^2 + y^2 + 2z^2 + 2zx + 2yz = 1$.

52. The nature of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$ is

<(co-2)>

a) Parabolic b) Hyperbolic c) Elliptic d) none

53. The integral surface of $yz_x + xz_y = x^2 + y^2$, passing through

$$x = 1 - t, y = 1 + t, z = 1 + t^2 \text{ is}$$

a) $z = xy + \frac{1}{2}(x^2 - y^2)^2$ b) $z = xy + \frac{1}{4}(x^2 - y^2)^2$

<(co-3)>

b) $z = xy + \frac{1}{8}(x^2 - y^2)^2$ d) $z = xy + \frac{1}{16}(x^2 - y^2)^2$

Answer Key:

1.a), 2. b), 3. b), 4. c), 5. c) , 6. a), 7. c) 8. a), 9. a), 10. a),11. b), 12. a),13. c), 14. d) , 15. d) 16.d), 17. a), 18. b), 19. a), 20. c) , 21. a), 22. a) 23. d), 24. b), 25. b), 26. d), 27. d),28. d), 29. d) , 30. a) 31.a), 32. a),33. d), 34. c), 35. b) , 36. c), 37. d) 38. a), 39. b), 40. d), 41. d), 42. c),43. b), 44. d) , 45. c) 46. a) 47. b) 48. a) 49. b) 50. d) 51. c) 52. b) 53. d)

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Long Question ; (Each question carries 3/ 5/7 marks)

1. Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle $x^2+y^2=1, z=0$. <Co-3> 5
2. Find the D'Alembert's solution of the infinite string problem $\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, $-\infty<x<\infty, t>0$, with $u(x,0)=f(x), u_t(x,0)=g(x)$, $-\infty<x<\infty$. <Co-3> 5
3. Solve onedimensional heat equation $\frac{\partial^2 u}{\partial x^2}=\frac{1}{\kappa}\frac{\partial u}{\partial t}$ with $u(0,t)=u(a,t)=0, \forall\, t>0$ and $u(x,0)=u_0$. <Co-4> 5
4. Solve $\frac{\partial^2 y}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ with $y(0,t)=0=y(l,t)$ and $(x,0)=\text{a}\sin\left(\frac{\pi x}{l}\right)$,$y_t(x,0)=0$. 5 <Co-3>
5. Reduce the equation $\frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 z}{\partial y^2}$to its canonical form. 5 <Co-3>
6. Show that the Cauchy Problem $2u_x+3u_y=5$,with $u=1$ on the line $3x-2y=0$ has no solution. <Co-3> 5
7. Reduce the equation $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$ to its canonical form. <Co-3> 5
8. Determine the solution of $u_{xx}-u_{yy}=1$ with $u(x,0)=sinx$ and$u_y(x,0)=x$. <Co-4> 5
9. Find the integral surface of the partial differential equation $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$ which contains the line $x+y=0, z=1$. 5 <Co-3>
10. Determine the solution $\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$,$0<x<l, t>0$ with $u(x,0)=\sin\left(\frac{\pi x}{l}\right), 0\le x\le l, u_t(x,0)=0, 0\le x\le l$ and the boundary condition $u(0,t)=0=u(l,t), t\ge 0$. <Co-3> 5
11. a) Apply Charpit's method to find the complete integral of the PDE
$$(p+q)(px+qy)=1$$
4 <Co-3>

    b) Form a PDE by eliminating the arbitrary function φ and ψ from the relation
$$u(x,y)=y\varphi(x)+x\psi(y)$$
<Co-3> 3
12. Using method of separation of variables solve the PDE $4z_x+z_y=3z$ under the condition $z=3e^{-y}-e^{-5y}$ at $x=0$. 7 <Co-3>
13. Using $\eta=x+y$ as one of the transformation variable. Obtain the canonical form of

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$u_{xx} - 2u_{xy} + u_{yy} = 0$ and hence solve it. 7 <CO-3>

14. A tightly stretched string of length l with fixed end points is initially at rest in its equilibrium position and each of its points is given a velocity v, which is given by

$$v(x) = \begin{cases} cx, 0 \le x < l/_2 \\ c(l-x), l/_2 \le x < l \end{cases}.$$
<CO-3>

Find the displacement. c being the wave speed. 7

15. Solve the following initial boundary value problem

$$u_t = u_{xx} (0 < x < \lambda, t > 0)$$

subject to the conditions <CO-3>

$$u(x,0) = 3\sin(n\pi x)\ (n, \text{a positive integer})$$

$$u(0,t) = u(\lambda,t) = 0\ .$$ 7

16. a) Apply Charpit's method to find the complete integral of the PDE <CO-3>

$$(px + qy) = pq$$ 4

c) Solve the partial differential equation

$$u_x{}^2 + u_y{}^2 = u$$ 3 <CO-3>

Using $u(x,y) = f(x) + g(y)$.

17. Reduce the second order PDE <CO-4>

$$yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$$

to a canonical form and hence solve it. 7

18. A stretched string of finite length L is fixed at its end and is subjected to an initial displacement $(x,0) = u_0 \sin\left(\frac{\pi x}{L}\right)$. The string is released from this position with zero initial velocity. Find the resultant motion of the string. 7 <CO-3>

19. a) Form the PDE by eliminating the arbitrary function f from the equation <CO-3>

$$z = xf\left(\frac{y}{x}\right)$$ 3

b) Find the solution of the equation <CO-3>

$$(y-u)u_x + (u-x)u_y = x - y$$ 5

With the Cauchy data $u = 0.$ on $xy = 1$.

20. Evaluate the integral $\iint \sqrt{4a^2 - x^2 - y^2}\,dxdy$ over E where E is the region bounded by the circle $x^2 + y^2 = 2ax$. 7 <CO-4>

21. By changing the order of integration, prove that $\int_0^1 dx \int_x^{\frac{1}{x}} \frac{ydy}{(1+xy)^2(1+y^2)} = \frac{\pi-4}{4}$. 7 <CO-4>

22. Show that the volume common to the cylinders

$x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ is $\frac{16a^3}{3}$. 7

<CO-4>

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Question Bank on Numerical Analysis

MTMA **SEM-VI** **Paper-CC -14**

MCQ (Each Question carries 2 marks)

(Each question below is followed by four possible answers of which exactly one is correct. Choose the correct answer with proper justification.)

1. If an equation can be written in the form $x = \varphi(x)$ then Iteration method is convergent if
   a) $|\varphi'(x)| \geq 1$ b) $|\varphi'(x)| = 1$ c) $|\varphi'(x)| \leq 1$ d) $|\varphi'(x)| < 1$ <CO-1>
2. Condition of convergence of Gauss Seidel method is
   a) $|a_{ii}| < \sum_{j=1,j\neq i}^{n}|a_{ij}|$ b) $|a_{ii}| \geq \sum_{j=1,j\neq i}^{n}|a_{ij}|$ c) $|a_{ii}| > \sum_{j=1,j\neq i}^{n}|a_{ij}|$ d) none
   $i = 1,2,\ldots\ldots\ldots,n$ <CO-1>
3. The order of convergence of Newton Raphson method is <CO-2>
   a) 2 b) 1 c) 3 d) 0
4. The order of convergence of Iteration method is <CO-2>
   b) linear b) quadratic c) cubic d) bi-quadratic
5. The degree of precision of Simpson's 1/3 rule is <CO-2>
   a) 1 b) 2 c) 3 d) 4
6. Sum of the Lagrangian coefficient is <CO-1>
   a) 0 b) 1 c) -1 d) 4
7. Lagrange's interpolation formula is used for
   a) Equally spaced interval b) unequal spaced interval c) if the interpolating point lies on the middle of the table d) all of the above <CO-1>
8. The condition of convergence of Newton Raphson method is
   a)$|f(x)f''(x)| \geq |f'(x)|^2$b)$|f(x)f''(x)| \leq |f'(x)|^2$c)
   $|f(x)f''(x)| > |f'(x)|^2$d)$|f(x)f''(x)| < |f'(x)|^2$ <CO-2>

9. Which of the following is correct ? <CO-2>

   a) $E \equiv 1 - \Delta$ b) $E \equiv 1 + \nabla$ c) $E \equiv 1 + \Delta$ d) $\Delta \equiv 1 + E$

10. Gauss Elimination method is

    a) direct method b) iterative method c) may be direct or iterative d) all of the above. <CO-1>

11. The order of convergence of Secant method is

<CO-1>

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b) 1.516 b) 1.616 c) 1.66 d) 2

12. The order of convergence of Regula falsi method is

a) 2 b) 1 c) 3 d) 0 <CO-1>

13. Gauss Seidel method is a

a) direct method b) iterative method c) may be direct or iterative d) all of the above. <CO-1>

14. The degree of precession of Weddle's rule is <CO-2>

a) 5 b) 2 c) 3 d) 4

15. The degree of precession of Trapezoidal rule is <CO-2>

a) 0 b) 1 c) 2 d) 3

16. Newton –Raphson method fails if <CO-1>

a) $f'(x) = 0$ b) $f''(x) \neq 0$ c) $f''(x) = 0$ d) $f'(x) \neq 0$

17. If the arguments are equally spaced and the interpolating point is beginning of the table, Then which formula gives better result

a) Newton's Forward b) Newton's backward c) Sterling's interpolation d) Bessel's interpolation formula <CO-1>

18. If the If the arguments are unequally spaced, then which formula is used

a) only Lagrange's interpolation b) only Newton's divided difference c) both Lagrange's interpolation and Newton's divided difference d) none. <CO-1>

19. How many iterations of the Bisection method must be required to guarantee that error is less than10^{-4} in the interval (0,2)? <CO-3>

a) 14 b) 15 c) 10 d) 12

20. Find the relative error in approximating $x = 2.301056$ by $x^* = 2.3112$ <CO-3>

a) 0.004408 b) 0.004389 c) 0.010144 d) none

21. If $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} \frac{|x_{n+1}-x|}{|x_n-x|^3} = \frac{1}{2}$, then the order of convergence of $\{x_n\}$ is <CO-3>

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a) $\frac{1}{2}$ b) 2 c) 3 d) $\frac{1}{3}$

22. Which of the following are closed root enclosing method to approximate the roots of a function <CO-2>

a) Newton-Raphson and Regula falsi b) Bisection and Secant c) Bisection and Regula falsi d) Secant and Regula falsi.

22. What is the order of convergence of Secant method?

a) 1.63 b) $\frac{1+\sqrt{5}}{2}$ c) 1.518 d) $1+\frac{\sqrt{5}}{2}$ <CO-1>

23. The Newton method is also called as

a) Tangent method b) Secant method c) Chord method d) Diameter method <CO-1>

24. Trapezoidal rule gives the exact value of the integral when the integrand is a <CO-1>

a) Linear function b) Quadratic function c) Cubic function d) Any polynomial.

25. Let $f(x) = x^2$. Find the 2[nd] order divided difference for the points x_0, x_1, x_2 <CO-3>

a) -1 b) $\frac{1}{x_0 - x_1}$ c) 1 d) $\frac{1}{x_2 - x_1}$.

26. Using the data given below, compute $\int_0^2 \{f(x)\}^2\,dx$ by Trapezoidal rule <CO-3>

x	0	1	2
$f(x)$	8	5	6

a) 92 b) 75 c) 123 d) 42.

27. Which of the following is an iterative method?

a) Gauss Jordan b) Gauss Seidel c) Gauss Elimination d) Factorization. <CO-1>

28. Which of the following is an assumption of Jacobi's method?

a) coefficient matrix has zeros on it's main diagonal b) coefficient matrix has no zeros on it's main diagonal c) iteration involved in Jacobi's method converges d) none. <CO-2>

29. Which of the following assumption are correct?

i) In Simpson's 1/3 rule requires the division of even number of sub intervals. <CO-2>

ii) In Simpson's 3/8 rule requires the division of number of sub intervals as a multiple of 6.

a) i)only b) ii)only c) both i) and ii) d) neither i) nor ii).

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30. Value of $E^{-1/2}$ is

a) $u-\frac{\delta}{2}$ b) $u+\frac{\delta}{2}$ c) $\frac{u}{2\delta}$ d) $\frac{\delta}{2u}$

<CO-2>

Answer:

1. c) 2. c) 3. a) 4. a) 5. c) 6. b) 7. d) 8. d) 9. c) 10. a) 11.b) 12.b) 13. b) 14. a) 15. b) 16. a) 17. a) 18. c) 19.b) 20.c) 21.c) 22.b) 23.a) 24. a) 25. c) 26. b) 27. b) 28. b)29. a) 30.a)

Long Questions (Each Question carries 3/5 marks)

1. What do you mean by eigen pair? Explain the Power method for finding dominant eigen pair. <CO-1>
2. Define divided difference . Derive divided differences formula for three arguments. <CO-1>
3. Establish Lagrange's polynomial interpolation formula(without error term).Is this polynomial unique? <CO-3>
4. Given that $\frac{dy}{dx}=1-\frac{y^2}{x}y(2)=2$ where $y(2)=2$. Compute $y(2.1)$ by Euler's Modified method correct to four decimal places taking $h=0.05$. <CO-3>
5. Describe Gauss-Seidel Iterative method for solving a system of linear equations, mentioning a stopping criterion. State also the condition of convergence of this method. <CO-3>
6. Find approximate solutions of the following initial value problem by Euler method and Runge-Kutta (4^{th} order) method at x=0.1 <CO-3>

   $\frac{dy}{dx}=x+y$ with $y(0)=1$ correct to four decimal places.
7. Deduce numerical differentiation formula (both 1^{st} and 2^{nd} order) from Newton's Forward interpolation formula mentioning at least three terms. Hence find the value of 1^{st} and 2^{nd} derivative at the left end point starting at least three terms. <CO-3>
8. Explain whether one can use Newton- Raphson method to find a real root If $f(x)=0$ has a multiple root? How it can be generalized and what will be its order of convergence. <CO-4>
9. Find the root of the equation $xe^x=cosx$ using Regula-Falsi method correct to three decimal places. <CO-3>
10. Solve the equation:
    $\frac{dy}{dx}=x^2+y^2, y(0)=1$, by fourth order Runge-Kutta method from$x=0$ to $x=0.2$, with step length h= 0.1. <CO-3>

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11. Integrate the Lagrange's interpolation polynomial
$\varphi(x) = \frac{x-x_0}{x_0-x_1} f(x_0) + \frac{x-x_1}{x_1-x_0} f(x_1)$ over the interval $[x_0, x_1]$ and establish the Trapezoidal rule. Give a comparison between Simpson's 1/3 rule and Simpson's 3/8 rule. <Co-3>

12. Write down the quadratic polynomial which takes the same value as $f(x)$ at $x = -1,0,1$ and integrate to obtain $\int_{-1}^{1} f(x)dx = \frac{1}{3}[f(-1) + 4f(0) + f(1)]$. <Co-3>
Assuming the error to have the form $Af^{iv}(\xi), -1 < \xi < 1$, find the value of A.

13. What do you mean by degree of precision of a mechanical quadrature formula? Prove that a necessary and sufficient condition for an (n+1) point quadrature formula with node set$\{x_0, x_1, x_2, \dots\dots\dots\dots, x_n\}$ to have degree of precision (2n+1) is that <Co-3>
$\int_a^b \omega(x)Q_n(x)dx = 0$ where $\omega(x) = (x - x_0)(x - x_1)(x - x_2) \dots\dots\dots\dots . (x - x_n)$
and $Q_n(x)$ is an arbitrary polynomial of degree $\leq n$.

14. a) Explain the method of Iteration for approximating a simple real root α of an equation of the form $x = \varphi(x)$ where $\varphi(x)$ and $\varphi'(x)$ are continuous in an interval about α. <Co-4>
    b) Derive a sufficient condition of convergence of the above method.
    c) Find also the order of convergence of the above method if $\varphi'(\alpha) = 0$.

15. Write down the basic assumptions for finding the dominant eigenvalue of a real matrixA_{nxn} by Power method. How the convergence rate of the method depends upon the magnitudes of its eigenvalues? State when the method fails. <Co-4>

16. Deduce Euler's method to solve initial value problem $y' = f(x,y), y(x_0) = y_0$ <Co-4>
Using Taylor's series expansion. Does this method always converge? Justify your answer.

17. Using Euler's modified method, solve the following differential equation <Co-3>
$\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$ for $x = 0.02$ by taking step length $h = 0.01$.

18. Using three successive approximations of Picard's method, obtain approximate solution of the differential equation$\frac{dy}{dx} = x^2 + y^2$ satisfying the initial condition $y(0) = 0$. <Co-3>

19. Describe Newton's method for solving a system of equations $f(x,y) = 0, g(x,y) = 0$ in two variables x and y .When does the method fail? <Co-2>

20. What is the condition of convergence of Gauss-Seidel method? Is it a necessary and sufficient condition? Compare this method with Gauss Elimination method. <Co-2>

21. Describe the Power method to calculate the numerically greatest eigenvalue of a real non-singular square matrix of order n. How do you find its numerically least eigenvalue? <Co-2>

22. a) What do you mean by the partial pivoting in solving of system of n linear equations in n unknowns? What are the reasons for such pivoting? <Co-1>

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b) Compute the total number of arithmetic operations(multiplications/divisions) in Gaussian algorithms for solving an nxn system of linear equations.

23. Show that if the iteration function of the equation $f(x) = 0$ is such that $|g'(x)| \leq k < 1$ for all x in $[a, b]$, then the sequence $\{x_n\}$ generated by $x_n = g(x_{n-1}); n = 1,2,3, \dots\dots$converges to the real root of $f(x) = 0$ uniquely for any choice of x_0 in $[a, b]$. <CO-3>

24. Show that the square root of $N = AB$ is given by $\sqrt{N} = \frac{S}{4} + \frac{N}{S}$, where $S = A + B$. <CO-3>

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QUESTION BANK

CC 6 : RING THEORY & LINEAR ALGEBRA -I

UNIT1 & UNIT2

Questions carrying 5 or 3 marks

1) Prove that a finite Integral Domain is a field . [CO 2]

2) Let (B, +, .) be a ring with unity such that $(a.b)^2 = a^2.b^2$ for all a ,b ϵ B , show that B is a commutative ring . [CO 3]

3) Find a solution of the equation ax = b in S_3 where [CO 3]

$a = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

4) Let S = { $a + b\sqrt{2}$: a , $b \in Q$ }. Prove that (S ,+, .) is a subfield of (ℝ,+, .). [CO 2]

5) Find the elements in Z_{12} which are zero divisors .

[CO 3]

6) Find the basis & dimension of the subspace W = {(x,y,z) ϵ R_3 : x+y+z = 0 }of R_3 [CO 3] .

7) Find the rank of the linear transformation T: $R^3 \rightarrow R^3$ defined by T(x,y,z) = (y , 0 , z). [CO 3]

8) Let V be the vector space of dimension n ,then find the dimension of its dual space. [CO 3]

9) Let V be the finite dimensional vector space and U & W are components to each other in V, Prove that dim V = dim U + dim W. [CO 3]

10) Find the basis & dimension of the subspace W of R^3 $where\ W = \{ (x, y, z):$ $x + 2y + 3z = 0 .\}$ [CO 3]

11) Let V be a vector space of dimension n over field F Then prove that any linearly independent set of n vectors of V is a basis of V. [CO 2]

12) Let A = $\begin{matrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{matrix}$ .Determine the Row space & Column Space of A.

[CO 3]

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QUESTION BANK

CC 6 : RING THEORY & LINEAR ALGEBRA -I

UNIT1 & UNIT2

Multiple Choice Questions (2marks)

1) A field F containing exactly 4elements has the characteristic
   i. 0
   ii. 2 [CO 3]
   iii. 3
   iv. 4

2) A subring of a field is
   i) An integral Domain
   ii) A Skew Field [CO 3]
   iii) A Field
   iv) None of the above

3) The number of conjugacy classes in the permutation group S_6 is
   b) 12 b)11 c) 10 d)6 [CO 3]

4) Upto isomorphism the number of abelian group of order 108 is
   b)12 b)9 c)6 d)5. [CO 3]

5) Let T : $R^6 \rightarrow R^6$ be a linear mapping such that $T^2 = O$ then the rank of T is
   a)≤ 3 $b) > 3$ $c)5$ $d)6$ [CO 3]
   .

6) The eigen values of an idempotent matrix are
   a) 0 , 1 b)1 , -1 c)0 , 0 d)1 , 1 [CO 3]
   .

7) Let F be a finite field of order q and nεN. then [GL(n ,F) : SL(n ,F)]is

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a)q b)q-1 c) q+1 d)q^n. [CO 3]

8) Let U = L {(2, 0 , 1) , (3, 0 , 1)} & W =L{(1,0 ,0), (0, 1, 0)} then dim (U∩V) is

a)1 b)2 c) 3 d) 0 [CO 3]

.

9) Let U = L {(1, 2, 1) , (2,1,3)} then dim U is [CO 3]

a) 1 b) 2 c) 3 d)none. of the above

10) The set S = {(2, 1 , 1) , (1, 2,1) , (1, 1,2)} then S is [CO 2]

a) Dependent inR^3 b) independent in R^3 c)a basis of R^3 d)none. of the above

11) The dimension of the subspace S = {(x ,y ,z) : 2x +y–z= 0 }is

a) 1 b) 2 c) 3 d) none. of the above [CO 3]

Question Bank

CC1: CALCULUS, GEOMETRY & VECTOR ANALYSIS

UNIT 1 (CALCULUS)

Questions carrying 5 marks

1. A semicubical parabola $ay^2 = x^3$, the tangent at any point P cuts the axis of y in M and the curve in Q . O is the origin & N is the foot of the ordinate of P . Prove that MN & OQ are equally inclined to the axis of x . [CO 3 , CO 2]

2. Show that $r = a\theta^n$ has point of inflexion iff n lies between 0 & -1 and that they are given by

   $\theta = \pm\sqrt{-n(n+1)}$. [CO 2]

3. A function f(x) is defined as follows : f(x) = x^3 in the interval -∞ $< x \leq 1$, $f(x) = ax^2 + bx + c$. In the interval 1<x<∞ . $What\ must\ be\ a\ ,b\ ,c\ be\ so\ that\ the\ curvature\ of\ the\ curve\ y = f(x)\ is\ continuous\ everywhere$. [CO 3]

4. Find the equation of the curve which has x = 0 , y = 0 , y = x , y = -x four asymptotes and which passes through (a , b) & which cuts its asymptotes in eight points that lie on a circle whose centre & radius a . [CO 3]

5. Find the area of the figure enclosed by the cardioide x = 2acost – acos2t , y = 2asint – asin2t . [CO 3]

6. Find the moment of inertia of a uniform rod of mass M & length 2a about an axis through one extremity of the rod & perpendicular to the rod . [CO 3]

7. Find the oblique asymptotes of the curve $x^3 - xy^2 + 2xy - 2x + y = 0$. [CO 3]

8. Find the area of the loop of the curve $r = 4(sin\theta)^2 cos\theta$. [CO 3]

Question Bank

CC1: CALCULUS, GEOMETRY & VECTOR ANALYSIS

UNIT 1 (CALCULUS)

Questions carrying 3 marks

1. Compute the area of the figure contained between the curve y = $\frac{1}{x^2+1}$& its asymptotes . [CO 3]
2. Show that for the curve given by r = f(θ) the curvature is given by (u+$\frac{d^2u}{d\theta^2}$)$sin^3\varphi$ where u = $\frac{1}{r}$. [CO 2]
3. State whether $\frac{d^2y}{dx^2} = 0$ always imply existence of point of inflexion of the curve y = x^4 . [CO 4]
4. Show that tangents drawn at the extremities of any chord of the cardioide r = a (1+cosθ) which passes through the pole are perpendicular to each other . [CO 2]
5. If the area of a loop of the curve r = asin3θ is mπa^2 , then find the value of m . . [CO 3]
6. Find the length of the loop of the curve x = t^2 , $y = t - \frac{1}{3}t^2$. [CO 3]

Question Bank

Multiple Choice Questions (2 marks)

A. The length of the polar subtangent for the curve r = a (1+cosθ) at $\theta = \frac{\pi}{2}$ is

i. $\frac{a}{2}$ ii. a . [CO 3]

iii. 2a iv. $\frac{3a}{2}$.

B. The least value of the radius of curvature of the curve x = 5t , y = 5 log sect is

i. 15 ii. 3 . [CO 3]

iii. 20 iv. 5 .

C. The envelope of the family of curves , $\mu x^2 + \mu^2 y = 1$, (μ *is a parmeter*)

i. $x^2 + 2y = 0$ ii. $x^4 + 4y = 0$

iii. $y^4 + 4x =$ iv. $y^2 + 2x = 0$ . . [CO 3]

D. The asymptotes of the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are

i. $\frac{x}{a} = \pm\frac{y}{b}$ ii. . $\frac{x}{b} = \pm\frac{y}{a}$. [CO 3]

iii. x= $\pm y$ iv . xy = $\pm ab$.

Question Bank

CC1: CALCULUS, GEOMETRY & VECTOR ANALYSIS

UNIT 2 (Geometry (Two-Dimension))

Questions carrying 5 / 4 / 3 marks

1. Show that the equation m($x^3 - 3xy^2) + y^3 - 3x^2y = 0$ represents three straight lines making equal angles with each other . . [CO 3]

2. Prove that the length of the focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which is inclined to the major axis at an angle θ is $\frac{2ab^2}{a^2(\sin\theta)^2 + b^2(\cos\theta)^2}$ . . [CO 2]

3. Show that the directrix of the parabola $x^2 + 2xy + y^2 - 4x + 8y - 6 = 0$ is $3x - 3y + 8 = 0$. [CO 2]

4. Show that the locus of the points of intersection of tangents to the parabola $y^2 = 4ax$ at points whose ordinates are in the ratio $p^2 : q^2$ is $y^2 = (\frac{p^2}{q^2} + \frac{q^2}{p^2} + 2)ax$. [CO 3]

5. Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ parallel to the tangent at θ=α , is given by

   $l^2(e^2 + 2e\cos\alpha + 1) = r(e^2 - 1)[\cos(\theta - \alpha) + e\cos\theta]$. [CO 2 ,CO3]

6. Examine whether the following conic is a central conic or not & find the nature of the conic $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$. [CO 4]

7. If the normal at two points μ & λ of a parabola intersect on the curve then show that $\lambda\mu = 2$. [CO 3]

8. Show that the equation of the straight line joining the feet of the perpendiculars from the points (d,0) on the straight lines $ax^2 + 2hxy + by^2 = 0$ *is* (a-b)x+2hy +bd=0 . [CO 2 , CO 3]

9. Show that the polar equation of any circle passing through the pole can be expressed in the form

   $r =$ Acos θ +Bsin θ ,where A & B are constants . [CO 2]

10. Show that the equation$(a^2 + 1)x^2 + 2(a + b)xy + (b^2 + 1)y^2$=c , (c>0) represents an ellipse of area $\frac{\pi c}{ab-1}$. [CO 3]

Multiple Choice Questions (2 marks)

1. The equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ represents
   a) a parabola b) an Ellipse c) a Hyperbola d) Pair of straight lines
2. The angle between the pair of lines represented by $4x^2 - 24xy + 11y^2 = 0$
   a) $tan^{-1}(4/3)$ b) $tan^{-1}(3/4)$ c)$tan^{-1}(1/2)$ d) $tan^{-1}(2)$
3. The equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ represents two parallel lines if
   a) $a = 1$ b) $a > 1$ c) $a < 1$ d) $d = 0$
4. The equation of the tangent to the conic $y^2 - xy - 2x^2 - 5y + x - 6 = 0$ at (1,-1) is
   a) $x + 4y - 3 = 0$ b) $x - 4y + 3 = 0$ c) $x - 4y - 3 = 0$ d) $x + 4y + 3 = 0$
5. The equation $\frac{8}{r} = 4 - 5cos\theta$ represents
   a) a Parabola b) an Ellipse c) a Hyperbola d) pair of Straight lines

Answer:

1.d) 2. a) 3. c) 4. d) 5. c)

Question Bank

CC1: CALCULUS , GEOMETRY & VECTOR ANALYSIS

UNIT 2 (Geometry (THREE-Dimension))

Questions carrying 5 / 3 marks

1. Show that the angle betweens the straight lines whose direction cosines are give by l+m+n=0 and fmn+gnl+hlm=0 is $\frac{\pi}{3}$, if $\frac{1}{f}+\frac{1}{g}+\frac{1}{h}$=0. [CO 2 , CO 3]

2. Find the equations of three planes through the points (3,1,1) and (1,-2,3) and parallel to the co-ordinate axis. [CO 3]

3. Find the image of the straight line $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-4}{2}$ $in\ the\ plane$ $2x-y-z+3=0.$ [CO 3]

4. A plane passes through a fixed point (a,b,c) and cuts the axes Ox, Oy, Oz in A, B, C resp. Show that the locus of the centre of the sphere OABC is
   $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$. [CO 3]

5. If the edges of a rectangular parallelepiped be a,b,c, show that the angles between the four diagonals are given by $\cos^{-1}\left(\frac{a^2\pm b^2\pm c^2}{a^2+b^2+c^2}\right)$. [CO 2]

6. Show that the equation $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ represents a pair of planes . [CO 2 , CO 3]

7. Show that the locus of a point which is equidistant from two given straight lines y = mx, z = c; y = $-mx$, z = $-c$ is $mxy+c(m^2+1)z=0$ [CO 2 , CO 3]

Question Bank

Multiple Choice Questions (2 marks)

1. The value of λ for which the planes $x+2y+\lambda z=9$ and $4x-3y+12z+13=0$ are perpendicular to each other
   a) $1/2$ b) 1 c) 0 d) $1/6$. [CO 3]

2. The two straight lines $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ are
   a) Coincident b) Skew c) Coplanar d) none [CO 3]

3. The distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane

   $x-y+z=5$ from the point (-1,-5,-10) is [CO 3]

   a) 10 units b) 11 units c) 12 units d) 13 units

4. The equation of the cone generated by the lines from origin to the meet of the circle through $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$ is
   a) $xy+yz+zx=0$
   b) $\left(\frac{b}{c}-\frac{c}{b}\right)yz+\left(\frac{c}{a}-\frac{a}{c}\right)zx+\left(\frac{a}{b}-\frac{b}{a}\right)xy=0$
   c) $\left(\frac{a}{b}+\frac{b}{c}\right)yz+\left(\frac{b}{c}+\frac{c}{a}\right)zx+\left(\frac{c}{a}+\frac{a}{b}\right)xy=0$ [CO 3]
   d) $\left(\frac{b}{c}+\frac{c}{b}\right)yz+\left(\frac{c}{a}+\frac{a}{c}\right)zx+\left(\frac{a}{b}+\frac{b}{a}\right)xy=0$

5. The equation of the sphere through origin and the points $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$ is
   a) $x^2+y^2+z^2$ +ax+ by +cz=0
   b) $x^2+y^2+z^2$ =ax+ by +cz [CO 3]

   c) $x^2+y^2+z^2=1$
   d) None

6. The equation of the tangent plane to the sphere $x^2+y^2+z^2=16$ at (1,-1,1) is
   a) $x-y+z=0$
   b) $x+y-z=16$
   c) $x-y-z=16$ [CO 3]
   d) $x-y+z=16$

7. The locus of luminous point of the Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that casts a parabolic shadow on the plane $z=0$

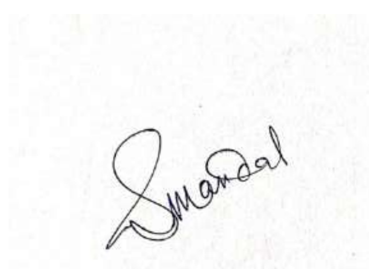
Multiple Choice Questions (2 marks)

1. The locus of luminous point of the Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that casts a parabolic shadow on the plane $z=0$

   a) $z=0$ b) $z=\pm c$ c) $x=\pm c$ d) $y=\pm c$
   [CO 3]

2. The quadric $2yz+2zx+2xy=1$ represents
   a) Right circular cone
   b) Hyperbolic Cylinder
   c) Hyperboloid of one sheet [CO 3, CO 4]
   d) Hyperboloid of two sheet

Answer:



1.d) 2. c) 3. d) 4. d) 5. b) 6. d) 7. b) 8. d)

Question Bank

CC1: CALCULUS, GEOMETRY & VECTOR ANALYSIS

UNIT -3 (VECTOR ANALYSIS)

Questions carrying 5 marks

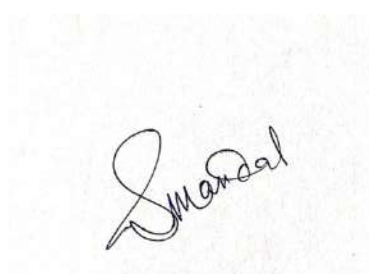
1) Show that the equations of a plane containing two parallel lines **r**= $\boldsymbol{a} + s\boldsymbol{b}$ and $\boldsymbol{r} = \acute{a} + t\boldsymbol{b}$ is **r.(a-á)×b=[a á b].** [CO 3]

2 Prove that the angle inscribed in a semicircle is a right angle . [CO 2]

3. If the diagonals of quadrilateral bisects each other then prove that the figure is a parallelogram. [CO 2]

4. If **a, b, c** be three non-coplaner vectors then show that **b×c, c×a, a×b** are also non-coplaner. [CO 3]

5. Prove that a necessary and sufficient condition for the vector function $\vec{a}$(t) to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$. [CO 2]



6. A particle moves along the curve $x = 2t^2\,, y = t^3 - 4t^2\,,\ z = 3t^2 - 5t$, find the components of velocity & acceleration at time t=1 in the direction of $\cdot 2\hat{\imath} + \hat{\jmath} - k$ [CO 3]

..

7. Find the greatest rate of increase of the function F = z^2xy at the point (1 , 0 , 3) . [CO 3]

8. Let F be a ttwice differentiable vector field . Find the value of the constant c such that curl F = 2xi -7cyj +5zk . [CO 3]

Questions carrying 3 marks

1. Prove that in a triangle ABC, [CO 2]
   $\cos(A + B) = \cos A \cos B - \sin A \sin B.$ [CO 2]
2. Show that **[b×c c×a a×b]**=$[\boldsymbol{a}\ \ \boldsymbol{b}\ \ \boldsymbol{c}]^{\mathbf{2}}$ [CO 2]
3. If $\vec{\alpha} = t^2\hat{\imath} - t\hat{\jmath} - (2t + 1)k$ & $\vec{\beta} = (2t - 3)\,\hat{\imath} + \hat{\jmath} - tk$ where $\hat{\imath}\,, \hat{\jmath}\,, k$ *have their usual meanings , find* $\frac{d}{dt}\,(\vec{\alpha} \times \frac{d\beta}{dt}\,)$. [CO 2]
4. For the curve $\vec{r} = (3t\,, 3t^2, 2t^3)$ show that $[\dot{r}\ \ddot{r}\ \dddot{r}] = 216$ [CO 2 , CO 3]
5. Find the directional derivative of ϕ = $xy^2z + 4x^2z$ at (-1 , 1 , 2) in the direction $\cdot 2\hat{\imath} + \hat{\jmath} - 2k$. [CO 3]

.

6. Find the unit normal to the surface $x^2 2y + 3yz = 4$ at the point (1,-1,2) which makes acute angle to the X axis [CO 3]

Multiple Choice Questions (2 marks)

Department of Mathematics, Shyampur Siddheswari Mahavidyalaya
CBCS; B.Sc. (H) Mathematics

1) If $\vec{a}\,,\vec{b}\,,\vec{c}$ *are unit coplanar vectors then the scalar triple product* $[\overrightarrow{2a-b}\ \ \overrightarrow{b-c}\ \ \overrightarrow{2c-a}]$ is

a) 0   b) 1   c) √3   d) -√3   [CO 3]

2) The value of
μ *such that the volume of the parallelopiped formed by the vectors* $(2\,,\mu\,,1)\,,(0\,,2\,,\mu\,),$

(1 , μ , 2) , $(\mu\ ,0\ ,2\ )$ becomes minimum is [CO 3]
a) $\frac{\sqrt{6}}{3}$ $b)\ \frac{\sqrt{2}}{3}$ $c)\ \frac{2}{3}$ $d)\ \sqrt{6}$.

3) let $\vec{a}\,,\vec{b}$ *be two non zero vectors such that* $|a+b| = |a-b|$ then

a) a is parallel to b   b) a is perpendicular to b   c) a = b   d) none .   [CO 3]

4) the vector $\vec{a} = (\mu^2-4)i\ +2j\ +(\mu^2-9)$k makes acute angle with

a) X axis for all values of μ b) Z axis for all values of μ c) y axis for all values of μ
d) all of the above .   [CO 3]

5) For the curve $\vec{r}\ = (\ 3t\ ,\ 3t^2\ ,\ 2t^3\ )$ *then* $[\dot{r}\ \ddot{r}\ \dddot{r}]$ is   [CO 3]

a) 0   b) 216   c) dependent on t   d) 128 .

6) The values of the constant a and b for which the vector field

F = (ycosx + axz , bsinx +2zy , x^2+y^2) is irrotational are [CO 3]

a) a=0 ,b=0   b) a=2 ,b=1   c) a=1 ,b=2   d) none .

7) consider two vector field F = (2x + y , x , 2z) & F’ = (siny , x , 0) then
a) F conservativre but F’ is not conservative . b) Both F& F’ are conservative . c) neither F nor F’ is conservative d) F not conservative but F’ is conservative .

[CO 3 , CO 4]

8) Let F be twice differentiable vector field . The value of the constant c such that curl F = (2xi -7cyj + 5zk) is [CO 3]

a) 0 b) 1 c) 7 d) 5.

9) The Laplacian of $\emptyset = sinkx\ sinly\ \exp\sqrt{k^2} + l^2 z$ is [CO 3]

a) Dependent on l & k b) non zero for all l & k c) independent on the values of k & l d) zero for finite set of values of k & l .

Question Bank

CC4 : GROUP THEORY - I

UNIT -1

Questions carrying 5 marks

1. Let (G,o) be a semigroup containing a finite number of elements in which both the cancellation laws hold. Prove that (G,o) is a group. Does this theorem hold for an infinite semigroup ? [CO 2 , CO 4]

2. Let (G,o) be a group and H, K be subgroups of (G,o). If $H \cup K$ is a subgroup of (G,o), then show that either $H \subset K$ or $K \subset H$. [CO 2]

3. Determine whether the set D of all odd integers forms a commutative group with respect to $*$ defined by $a * b = a+b-1$ for $a,b \in D$. [CO 3]

4. Let S be a set of n elements. How many different binary compositions can be defined on S? How many different commutative binary compositions can be defined on S? [CO 3 , CO 4]

5. Let (S,o) be a semigroup. If for $x , y \in S$, $x^2 oy = y = yox^2$, prove that (S,o) is an abelian group. [CO 3]

Question Bank

CC4 : GROUP THEORY - I
UNIT -1

Questions carrying 3 marks

6. Find all elements of order 8 in the group (Z_{24} , +). [CO 3]

7. In a group G , a and b are distinct elements of order 2. If a and b commute, prove that o(ab) = 2. [CO 3]

8. In a group (G,o), a is an element of order 30. Find the order of a^{18}. [CO 3]

9. Find the units and the idempotent elements in the monoid (Z_6,.). [CO 3]

10. Show that (Z,-) is a quasi group but not a semigroup. [CO 2]

Multiple Choice Questions (2 marks)

1. If every element of a group be its own inverse, then the group is [CO 3]
   i) infinite ii) non-commutative
   iii) commutative iv) none of these

2. The order of an element in a group G is 30. Then o(a^{18}) is equal to [CO 3]
   i) 3 ii) 6 iii) 8 iv) 5

Question Bank

CC4 : GROUP THEORY - I

UNIT -1

3. Let G be a commutative group. Suppose G has subgroups of order 45 and 75. If $o(G)<400$, then $o(G)=$
   i) 90 ii) 150 iii) 225 iv) none of these [CO 3]

4. In an abelian group, if $o(a)=5$ and $o(b)=7$, then $(ab)^{14}$ is equal to
   i) a ii) a^{-1} iii) ab iv) b [CO 3]

5. Let H and K be two subgroups of a group G such that H has 7 elements and K has 13 elements. Then the number of elements of HK is
   i) 1 ii) 5 iii) 0 iv) 91 [CO 3]

6. In a group, the number of idempotent element(s) is
   i) 1 ii) 2 iii) 3 iv) infinite [CO 3]

7. All non-trivial subgroups of (Z,+) are
   i) finite groups
   ii) infinite groups [CO 4]
   iii) non-commutative
   iv) none of these

8. In Klein 4-group, the order of each non-identity element is
   i) 2 ii) either 2 or 4 [CO 3]
   iii) 4 iv) neither 2 nor 4

Answer: 1.iii) 2.iv) 3.iii) 4.ii) 5.iv) 6.i) 7.ii) 8.i)

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Question Bank

CC4 : GROUP THEORY - I

UNIT -2

Questions carrying 5 or 3 marks

1) Prove or disprove : Every group of order $\leq$ 5 is cyclic . [CO 3] , [CO 2]

2) In an abelian group G of order 10 contains an element of order 5 , prove that G must be a cyclic group . [CO 3]
3) Prove that if in a group if every elements has its own inverse , then the group is abelian . [CO 3]

4) State & prove Lagrange Theorem [CO 2]

.

5) State & Prove Little Fermat's Theorem . [CO 2]

Multiple Choice Questions (2 marks)

1) The generators of the cyclic group S = { 1 , -1 , i , -i }

   i. -1 , -i [CO 3,]
   ii. i , -i
   iii. 1 , -1
   iv. 1 , -1 .

2) Let G be a non abelian group . Then the order of G can be [CO 3]

Question Bank

CC4 : GROUP THEORY - I

UNIT -2

a) 25 b) 35 c) 121 d) 125
.

3) The number of conjugacy classes in the permutation group S_6 is [CO 3]
   a) 12 b) 11 c) 10 d) 6 .

4) Upto isomorphism the number of abelian group of order 108 is
   a) 12 b) 9 c) 6 d) 5 . [CO 3, CO4]

5) The number of permutations of order 6 in the group s_3 are [CO 3]
   a) 10 b) 12 c) 20 d) none .

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Question Bank

CC 4: GROUP THEORY-I

UNIT 3

Questions carrying 5 or 3 marks

1) Prove that the centre Z(G) is a normal subgroup of a group G .
   [CO 2]
2) What do you mean by conjugate of a subgroup , Prove that the conjugate subgroup aHa^{-1} is a normal subgroup of H . [CO 1 , CO 2]

3) Let G be a group of order 10 having a normal subgroup of order 2 , prove that G is commutative . [CO 3]

4) Let G be a group and $a \epsilon G$ prove that $\langle a \rangle$ is a normal subgroup of $C\langle a \rangle$. [CO 3]

Multiple Choice Questions (2 marks)

1) Let G be a non commutative group of order 10 , then the centre of G is [CO 4]
   a) Trivial b) non trivial c) G itself d) none .
2) Let m be a positive integer , then $\frac{Q}{Z}$ has [CO 4]
   a) Cyclic subgroup of order m b) non cyclic group of order m c) no subgroup of order m d) none .
3) The number of homomorphisms from the group Z_4 to Z_6 are [CO 4]
   a) 1 b) 2 c) 3 d) none .
4) A finite cyclic group of order n is isomorphic to [CO 3]
   a) Z_m $b)$ Z_{mn} $c)$ Z_n d) none .

QUESTION BANK

CC 11: PROBABILITY & STATISTICS

UNIT 1 , UNIT 2 & UNIT 3 (PROBABILITY)

Questions carrying 5 or 3 marks

1. What do you mean by mutual independence of n events $A_{1\,,}A_{2\,,}\,\dots\dots\dots\dots, A_n$.(n>2) [CO 2]
2. The probability of hitting a target is .001 , for each shot . Find the probability of hitting the target with two or more bullets if the number is 5000 . [CO 3]
3. Find the minimum number of times a fair die has to be thrown such that the probability of no six is less than 0.5 . [CO 3]
4. Evaluate the distribution function of the following distribution : Spectrum of the random variable X is {-1 , 0 , 2 , 3}with $P(X=-1)=\frac{1}{7}$, $P(X=0)=\frac{2}{7}$, $P(X=2)=\frac{3}{7}$, $P(X=3)=\frac{1}{7}$. [CO 3]
5. State & Prove Bayes theorem . [CO 2]
6. State & Prove Tchybechebs inequality . [CO 2]
7. An unbiased coin is tossed 100 times . then find r when the probability of finding r heads in 10 tosses is maximum . [CO 3]
8. Let the probability distribution function of X is F(x) = $-e^{-x}$, $0 \leq x\, \infty$

   Then find the probability density function of X . [CO 3]

Multiple Choice Questions (2 marks)

1) The value of skewness of asymmetrical distribution is
   a) -1 b) 1 c) 0 d) none . [CO 3]
2) For normal (m , σ) distribution which of the following is true
   a) m> 0 , $\sigma > 0$ b) $m < 0$,$\sigma > 0$ c) $m > 0$,$\sigma < 0$ d) none .
   [CO 3 , CO 2]
3) Let S denotes the sum of points obtained when two fair dice are rolled together , the Variance of S is
   a) $\frac{35}{3}$ b) $\frac{35}{6}$ c) $\frac{35}{12}$ d) $(\frac{35}{12})^2$ [CO 3]

QUESTION BANK

CC 11: PROBABILITY & STATISTICS

4) Let X be a non negative integer valued random variable with E(X) =1 then $\sum_{i=1}^{\infty}(P(X \geq i))$ is
   a) 0 b) 2 c) 1 d) none . [CO 3]
5) The number of elements of the largest σ field over the sample space where in a coin is tossed once is
   a) 4 b) 8 c) 16 d) 2. [CO 3]
6) The second central moment of the Bionomial distribution b(1 , 0.5) is
   a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) 1 . [CO 3]
7) A biased coin is tossed 4 times or untiil a head turn up , which ever occur earlier . the distribution of the number of tails turnin gup is [CO 3 , CO 4]
   a) Bionomial b) Geometric c) Negetive Bionomial d) none .
8) The value of P(X > 1) , where the p.d.f is given by f(x) = $\begin{cases} 1-\frac{1}{2}e^{-x} & , x \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$.

   a) $\frac{2}{e}$ b) $\frac{1}{e}$ c) $\frac{4}{e^4}$ d) $\frac{1}{2e}$. [CO 3]

QUESTION BANK

CC 11: PROBABILITY & STATISTICS

UNIT 4 & UNIT 5 (STATISTICS)

Questions carrying 5 or 3 marks

1) Let T_1 & T_2 be two statistics with E(T_1) $= (\theta_1 + \theta_2)$ E(T_2) $= (\theta_1 - \theta_2)$, then find the unbiased estimator of $\frac{T_1 - T_2}{2}$.
[CO 3 , CO 4]
2) Let X has a uniform distribution in (0 , 4) . A random variable Y is defined by
Y = 1 , $0 < X < 3$

2 , $3 \leq X < 4$, then Var (Y) . [CO 3]
3) Find the mean of the largest observation in a sample of size n drawn without replacement from a population N cards 1 , 2, 3 , ...N . [CO 3]
4) Let S_n denotes the no. of times face 5 appears in the independent rolling of balanced dice . then find $\lim_{n \to \infty} \left(\frac{6S_n - n}{\sqrt{5n}} \right) \geq 0$. [CO 3]

Multiple Choice Questions (2 marks)

1) The value of skewness of a symmetrical distribution is
b) -1 b) 1 c) 0 d) none . [CO 2]
.

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2) For a population of sample size n having variance σ^2 the variance of sample mean is

   a) $\frac{\sigma}{\sqrt{n}}$ $b)$ $\frac{\sigma}{n^2}$ $c)$ $\frac{\sigma^2}{n}$ $d)$ $\frac{\sigma^2}{n^2}$ [CO 3]

3) As we increase the sample size of a random sample the standard error of the mean

   a) Decreases b) increases c) remains same d) none . [CO 2]

QUESTION BANK

CC 11: PROBABILITY & STATISTICS

UNIT 4 & UNIT 5 (STATISTICS)

4) As we increase the sample size for a random sample , the shape of the sampling distribution of the mean is [CO 2]

   a) becomes more wide & flat b) remains same as it is c) becomes more skewed d) none .

5) in sampling theory 'SRSWR' means [CO 2]

   a) simple random sampling with replacement b) simple random sampling without replacement c) simple random sampling with reasoning . d) simple random sampling without reasoning . [CO 2]

6) for what value of μ , the following will be the incident matrix of a BIRD

$$N = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & \mu \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

[CO 3]

   a) $\mu = 0$ b) $\mu = 4$ c) $\mu = 3$ d) $\mu = 1$

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QUESTION BANK

<u>CC 10: MECHANICS</u>

<u>UNIT 1 & UNIT 2</u>

Questions carrying 5 or 3 marks

1. A particle rests on a rough curve whose equation is f(x , y) = 0 & is acted upon by forces the sums of whose components along the axes of x and y are X and Y resp. Prove that the particle will rest in equilibrium at those points for which (Xf_x + Yf_y)> $\left[(X^2+Y^2)({f_x}^2+{f_y}^2)^{\frac{1}{2}}\right]$, where λ *is the* angle of friction . [CO 3 , CO 2]

2. Show that three coplanar forces P , Q , R acting at the points A , B , C are in astatic equilibrium if they meet at a point on the circumcircle of the triangle ABC and if P : Q : R = a : b : c where

   a , b , c are the sides of the triangle ABC . [CO 3]

3. The altitude of a cone is h & the radius of its base is r ; a string is fastened to the vertex & to a point on the circumference of the circular base and is then put over a smooth peg ; show that the cone rests with its axis horizontal , the length of the string must be $\sqrt{h^2+4r^2}$. [CO 3 , CO 4]

4. A hemispherical shell rests on a rough inclined plane whose angle of friction is ; show that the inclination of the plane base of the rim to the horizon cannot be greater than $\sin^{-1}(2\sin\lambda)$. [CO 3]

5. Three forces P , Q , R act along the sides of a triangle formed by the lines x+y = 3 , 2x+y = 1 , & x-y = -1 . Find the equation of the line of action . [CO 3]

6. If a variable system of forces in a plane have constant moments about two fixed points in the plane , prove that the resultant passes through another fixed point

   . [CO 3]

QUESTION BANK

CC 10: MECHANICS

UNIT 3, UNIT 4 & UNIT 5

Questions carrying 5 marks

1. A particle is executing S.H.M of amplitude a , under an attraction $\frac{\mu x}{a}$. If a small disturbing force $\frac{v x^3}{a^3}$ towards the centre be introduced (the amplitude being unchanged) , show that the period is , to a first approximation decreased in the ratio $\left(1 - \frac{3v}{8\mu}\right) : 1$.

   [CO 3]
2. Two points P , Q are describing concentric circles of radii 'a' & 'b' and centre O , with velocities 'u' and 'v' . Find the velocity of P relative to Q when the angle POQ is θ and if the angular velocity of one relative to the other is zero , then $\cos\theta = \frac{au+bv}{av+bu}$.

   [CO 3 , CO 4]

3. The axes of x and y are along horizontal and vertical , a particle is projected from the origin O with a velocity 'u' . Prove that the conditions that the particle passes through A(h,k) is
   $u^2 \geq g\left[k + \sqrt{h^2 + k^2}\right]$. (the resistance of air is neglected)

   [CO 3 , CO 2]

4. A particle of mass m is projected vertically under gravity , the resistance of air being mk times the velocity ; show that the greatest height attained by the particle is $\frac{v^2}{g}[\lambda - \log(1+\lambda)]$, where V is the terminal velocity of the particle and λV is its initial vertical velocity . [CO 3 , CO 2]

QUESTION BANK

CC 10: MECHANICS

UNIT 3, UNIT 4 & UNIT 5

Questions carrying 3 marks

1. A particle is describes the conic $ax^2 + by^2 = c$ under the action of a force parallel to the force must vary as y^{-3} . [CO 3 , CO 2]

2. Show that , in elliptic motion about a focus under attraction μr^{-2} the radial velocity is given by the equation $r^2\left(\frac{dr}{dt}\right)^2 = \frac{\mu}{a}\{a(1+e) - r\}\{r - a(1-e)\}$.
[CO 3 , CO 2]
3. A particle of mass 'm' falls from rests towards a centre of force varying inversely as the square of the distance from the centre . Show that the time of descend through the first half of its initial distance is to that through the last half as $(\pi + 2)$: $(\pi - 2)$.
[CO 3 , CO 4]
4. A stone is dropped from a certain height and is observed to full the last 'h' cm. in 't' sec. Show that the total time of fall is $(\frac{t}{2} + \frac{h}{gt})$ sec . [CO 3 ,]

Multiple Choice Questions (2 marks)

1) A particle falls from rest under gravity whose resistance is k.v^2, the terminal speed is

i. $\sqrt{\frac{g}{k}}$ ii. $\frac{k}{g}$ [CO 3 , CO 4]

iii. $\sqrt{\frac{k}{g}}$ iv. $\frac{g}{k}$.

2) If the radial velocity is proportional to the transverse velocity , the path of the particle is

i. Circle ii. Catenary [CO 3 , CO 4]

iii. Cardioide iv. Parabola .

3) If the tangential & normal acceleration of a particle moving in a plane curve are equal , the velocity is given by v =

i. ce^{ψ} ii. c$e^{-\psi}$ [CO 3 ,]

iii. c$e^{2\psi}$ iv. c$e^{-2\psi}$

4) At what height would the kinetic energy of a falling particle be equal to half of its potential energy

a) $\frac{h}{2}$ b) $\frac{2h}{3}$ c) $\frac{h}{3}$ d) none . [CO 3]

5) A force F = 7-3x +$4x^2$ N acts on a body of one kg and displaces it from x = 0 to x = 4 them the work done is

a) 196 b) 188 c) 80 d) 192 . [CO 3 , CO 4]

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